Department of Mathematical Sciences Courses

About Course Numbers:

Each Carnegie Mellon course number begins with a two-digit prefix that designates the department offering the course (i.e., 76-xxx courses are offered by the Department of English). Although each department maintains its own course numbering practices, typically, the first digit after the prefix indicates the class level: xx-1xx courses are freshmen-level, xx-2xx courses are sophomore level, etc. Depending on the department, xx-6xx courses may be either undergraduate senior-level or graduate-level, and xx-7xx courses and higher are graduate-level. Consult the Schedule of Classes (https://enr-apps.as.cmu.edu/open/SOC/SOCServlet/) *each semester for course offerings and for any necessary pre-requisites or co-requisites.*

21-020 Calculus Companion Seminar

Fall: 2 units
This is a companion course designed to run alongside 21-120 Differential and Integral Calculus in the fall semesters. Students in this course will be presented with precalculus topics that are necessary for success in 21-120. Students must also be enrolled in 21-120 in order to register for this course.

21-101 Freshman Mathematics Seminar

Intermittent: 3 units
This course is offered in the Fall semester for first semester Freshmen interested in majoring in mathematics. Topics vary from year to year. Recent topics have included Fermat's last theorem, finite difference equations, convexity, and fractals. (Three 50 minute lectures)

21-102 Exploring Modern Mathematics

Fall and Spring: 9 units
This course is designed for non-math majors who are interested in learning some contemporary applications of mathematics with minimal prerequisite math knowledge. The course will survey the mathematical concepts centered along various themes, which may include the mathematics of social choice (voting and apportionment systems), topics in management science (optimization and elementary graph theory), modeling growth systems (population and finance), shape and form (symmetry and fractals), basic applications of probability and counting, and basic applications of number theory (cryptography and coding theory). Additional topics may be presented at the discretion of the instructor.

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-108 Introduction to Mathematical Concepts

Fall and Spring: 6 units
This course is an introduction to the vocabulary necessary for understanding and proving mathematical statements. The topics in this course include integers, rational numbers, polynomials, divisibility of numbers and polynomials, basic logic, sets, relations, functions, rule of sum, and rule of product. (Three 50 minute lectures, two 50 minute recitations)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-111 Calculus I

Fall: 10 units
Review of basic algebra, functions, limits, derivatives of algebraic, exponential and logarithmic functions, curve sketching, maximum-minimum problems. Successful completion of 21-111 and 21-112 entitles a student to enroll in any mathematics course for which 21-120 is a prerequisite. (Three 50 minute lectures, two 50 minute recitations)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-112 Calculus II

Fall and Spring: 10 units
Definite and indefinite integrals, and hyperbolic functions; applications of integration, integration by substitution and by parts. Successful completion of 21-111 and 21-112 entitles a student to enroll in any mathematics course for which 21-120 is a prerequisite. (Three 50 minute lectures, two 50 minute recitations)
Prerequisite: 21-111

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-120 Differential and Integral Calculus

All Semesters: 10 units
Functions, limits, derivatives, logarithmic, exponential, and trigonometric functions, inverse functions; L'Hospital's Rule, curve sketching, Mean Value Theorem, related rates, linear and approximations, maximum-minimum problems, inverse functions, definite and indefinite integrals; integration by substitution and by parts. Applications of integration, as time permits. (Three 50 minute lectures, two 50 minute recitations)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-122 Integration and Approximation

All Semesters: 10 units
Integration by trigonometric substitution and partial fractions; arclength; improper integrals; Simpson's and Trapezoidal Rules for numerical integration; separable differential equations, Newton's method, Euler's method, Taylor's Theorem, including a discussion of the remainder, sequences, series, power series. Parametric curves, polar coordinates, vectors, dot product. (Three 50 minute lectures, two 50 minute recitations)
Prerequisites: 21-112 or 21-120

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-124 Calculus II for Biologists and Chemists

Spring: 10 units
This is intended as a second calculus course for biology and chemistry majors. It uses a variety of computational techniques based around the use of MATLAB or a similar system. Topics to be covered include: Integration: techniques and numerical integration. Ordinary differential equations: techniques for solving ODEs and numerical methods. Modeling with ODEs (e.g., infection, population models). Linear algebra: matrices, complex numbers, eigenvalues, eigenvectors. Systems of ordinary differential equations (if time allows: stability of differential systems). Probability: discrete and continuum probability, conditional probability and independence, limit theorems, important distributions, probabilistic models. (Three 50 minute lectures, two 50 minute recitations)
Prerequisites: 21-120 or 21-112

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-127 Concepts of Mathematics

All Semesters: 12 units
This course introduces the basic concepts, ideas and tools involved in doing mathematics. As such, its main focus is on presenting informal logic, and the methods of mathematical proof. These subjects are closely related to the application of mathematics in many areas, particularly computer science. Topics discussed include a basic introduction to elementary number theory, induction, the algebra of sets, relations, equivalence relations, congruences, partitions, and functions, including injections, surjections, and bijections. A basic introduction to the real numbers, rational and irrational numbers. Supremum and infimum of a set. (Three 50 minute lectures, two 50 minute recitations)
Prerequisites: 21-112 Min. grade C or 15-112 Min. grade C or 21-120 Min. grade C or 21-108 Min. grade C

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-128 Mathematical Concepts and Proofs

Fall: 12 units
This course is intended for MCS first-semester students who are interested in pursuing a major in mathematical sciences. The course introduces the basic concepts, ideas and tools involved in doing mathematics. As such, its main focus in on presenting informal logic, and the methods of mathematical proof. These subjects are closely related to the application of mathematics in many areas, particularly computer science. Topics discussed include a basic introduction to elementary number theory, induction, the algebra of sets, relations, equivalence relations, congruences, partitions, and functions, including injections, surjections, and bijections. A basic introduction to the real numbers, rational and irrational numbers. Supremum and infimum of a set. (Three 50 minute lectures, two 50 minute recitations)
Prerequisites: 15-112 Min. grade C or 21-112 Min. grade C or 21-120 Min. grade C or 21-108 Min. grade C

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-150 Mathematics and the Arts

Intermittent: 9 units
Mathematics and the creative arts have long, interlinked histories. This course touches upon a broad range of mathematical ideas and the writers, artists, and art movements that were influenced and inspired by them. Topics include the use of geometric patterns in Islamic art, the influence of non-Euclidean geometry on Cubism, the constrained writing experiments of the Oulipo, and literary works exploring the concept of infinity.

21-201 Undergraduate Colloquium

Fall and Spring: 1 unit
The purpose of this course is to introduce math majors to the different degree programs in Mathematical Sciences, and to inform math majors about relevant topics such as advising, math courses, graduate schools, and typical career paths in the mathematical sciences. The Career and Professional Development Center will present modules on professional communication, developing interview and networking skills, and preparing for career fairs. (One 50 minute session)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-228 Discrete Mathematics

Fall and Spring: 9 units
The techniques of discrete mathematics arise in every application of mathematics, which is not purely continuous, for example in computer science, economics, and general problems of optimization. This course introduces two of the fundamental areas of discrete mathematics: enumeration and graph theory. The introduction to enumeration includes permutations, combinations, and topics such as discrete probability, combinatorial distributions, recurrence relations, generating functions, Ramsey's Theorem, and the principle of inclusion and exclusion. The introduction to graph theory includes topics such as paths, walks, connectivity, Eulerian and Hamilton cycles, planar graphs, Euler's Theorem, graph coloring, matchings, networks, and trees. (Three 50 minute lectures, one 50 minute recitation)
Prerequisites: 21-128 or 15-151 or 21-127

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-235 Mathematical Studies Analysis I

Fall: 12 units
A component of the honors program, 21-235 is a more demanding version of 21-355 of greater scope. Topics to be covered typically include: metric spaces, normed spaces, and inner product spaces; further properties of metric spaces such as completions, density, compactness, and connectedness; limits and continuity of maps between metric spaces, homeomorphisms, extension theorems, contraction mappings, extreme and intermediate value theorems; convergence of sequences and series of functions; metric spaces of functions, sequences, and metric subsets; Stone-Weierstrass and Arzela-Ascoli theorems; Baire category and applications; infinite series in normed spaces, convergence tests, and power series; differential calculus of maps between normed spaces, inverse and implicit function theorems in Banach spaces; existence results in ordinary differential equations. The prerequisite sequence 21-128, 21-242, 21-269 is particularly recommended. (Three 50 minute lectures, one 50 minute recitation)
Prerequisites: 21-256 or 21-268 or 21-269 or 21-259

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-236 Mathematical Studies Analysis II

Spring: 12 units
A component of the honors program, 21-236 is a more demanding version of 21-356 of greater scope. Topics to be covered typically include: Lebesgue measure in Euclidean space, measurable functions, the Lebesgue integral, integral limit theorems, Fubini-Tonelli theorem, and change of variables; Lebesgue spaces, completeness, approximation, and embeddings; absolutely continuous functions, functions of bounded variation, and curve lengths; differentiable submanifolds of Euclidean space, tangent spaces, mappings between manifolds, vector and tensor fields, manifolds with boundary and orientations; differential forms, integration of forms, Stokes' theorem; Hausdorff measure, divergence theorem. (Three 50 minute lectures, one 50 minute recitation)
Prerequisite: 21-235 Min. grade B

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-237 Mathematical Studies Algebra I

Fall: 12 units
A component of the honors program, 21-237 is a more demanding version of 21-373 (Algebraic Structures) of greater scope. Abstract algebra is the study of algebraic systems by the axiomatic method, and it is one of the core areas of modern mathematics. This course is a rigorous and fast-paced introduction to the basic objects in abstract algebra, focusing on groups and rings. Group-theoretic topics to be covered include: homomorphisms, subgroups, cosets, Lagrange's theorem, conjugation, normal subgroups, quotient groups, isomorphism theorems, automorphism groups, characteristic subgroups, group actions, Cauchy's theorem, Sylow's theorem, normalisers, centralisers, class equation, finite p-groups, permutation and alternating groups, direct and semidirect products, simple groups, subnormal series, the Jordan-Holder theorem. Ring-theoretic topics include: subrings, ideals, quotient rings, isomorphism theorems, polynomial rings, Zorn's Lemma, prime and maximal ideals, prime and irreducible elements, factorization, PIDs and UFDs, Noetherian domains, the Hilbert Basis Theorem, Gauss' lemma and the Eisenstein criterion for irreducibility, fields of fractions, properties of polynomial rings over fields and UFDs, finite fields and applications. The prerequisite sequence of 21-128, 21-242, 21-269 is particularly recommended. (Three 50 minute lectures, one 50 minute recitation)
Prerequisites: (21-128 or 21-127 or 15-151) and (21-269 or 21-268) and (21-241 or 21-242)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-238 Mathematical Studies Algebra II

Spring: 12 units
A component of the honors program, 21-238 is a more demanding version of 21-341 (Linear Algebra) of greater scope. Linear algebra is a crucial tool in pure and applied mathematics. This course aims to introduce the main ideas at a high level of rigour and generality. The course covers vector spaces over arbitrary fields and the natural generalization to modules over rings. Vector space topics to be covered include: fields, Zorn's Lemma, vector spaces (possibly infinite dimensional) over an arbitrary field, independent sets, bases, existence of a basis, exchange lemma, dimension. Linear transformations, dual space, multilinear maps, tensor products, exterior powers, the determinant, eigenvalues, eigenvectors, characteristic and minimal polynomial of a transformation, the Cayley-Hamilton theorem. Module-theoretic topics to be covered include: review of (commutative) rings, R-modules, sums and quotients of modules, free modules, the structure theorem for finitely generated modules over a PID, Jordan and rational canonical forms, structure theory of finitely generated abelian groups. Further topics in real and complex inner product spaces include: orthonormal sets, orthonormal bases, the Gram-Schmidt process, symmetric/Hermitian operators, orthogonal/unitary operators, the spectral theorem, quadratic forms, the singular value decomposition. Possible additional topics: applications to combinatorics, category theory, representations of finite groups, unitary representations of infinite groups. (Three 50 minute lectures, one 50 minute recitation)
Prerequisite: 21-237 Min. grade B

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-240 Matrix Algebra with Applications

Fall and Spring: 10 units
Vectors and matrices, the solution of linear systems of equations, vector spaces and subspaces, orthogonality, determinants, real and complex eigenvalues and eigenvectors, linear transformations. The course is intended for students in Economics, Statistics, Information Systems, and it will focus on topics relevant to these fields. (Three 50 minute lectures, one 50 minute recitation)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-241 Matrices and Linear Transformations

All Semesters: 11 units
A first course in linear algebra intended for scientists, engineers, mathematicians and computer scientists. Students will be required to write some straightforward proofs. Topics to be covered: complex numbers, real and complex vectors and matrices, rowspace and columnspace of a matrix, rank and nullity, solving linear systems by row reduction of a matrix, inverse matrices and determinants, change of basis, linear transformations, inner product of vectors, orthonormal bases and the Gram-Schmidt process, eigenvectors and eigenvalues, diagonalization of a matrix, symmetric and orthogonal matrices. 21-127 is strongly recommended. (Three 50 minute lectures, two 50 minute recitations)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-242 Matrix Theory

Fall: 11 units
A component of the honors program, 21-242 is a more demanding version of 21-241 (Matrix Algebra and Linear Transformations), of greater scope, with increased emphasis placed on rigorous proofs. Topics to be covered: complex numbers, real and complex vectors and matrices, rowspace and columnspace of a matrix, rank and nullity, solving linear systems by row reduction of a matrix, inverse matrices and determinants, change of basis, linear transformations, inner product of vectors, orthonormal bases and the Gram-Schmidt process, eigenvectors and eigenvalues, diagonalization of a matrix, symmetric and orthogonal matrices, hermitian and unitary matrices, quadratic forms. (Three 50 minute lectures, two 50 minute recitations)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-254 Linear Algebra and Vector Calculus for Engineers

Fall and Spring: 11 units
This course will introduce the fundamentals of vector calculus and linear algebra. The topics include vector and matrix operations, determinants, linear systems, matrix eigenvalue problems, vector differential calculus including gradient, divergence, curl, and vector integral calculus including line, surface, and volume integral theorems. Lecture and assignments will emphasize the applications of these topics to engineering problems. (Three 50 minute lectures, one 50 minute recitation)
Prerequisite: 21-122

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-256 Multivariate Analysis

Fall and Spring: 9 units
This course is designed for students in Economics or Business Administration. Matrix algebra: vectors, matrices, systems of equations, dot product, cross product, lines and planes. Optimization: partial derivatives, the chain rule, gradient, unconstrained optimization, constrained optimization (Lagrange multipliers and the Kuhn-Tucker Theorem). Improper integrals. Multiple integration: iterated integrals, probability applications, triple integrals, change of variables. (Three 50 minute lectures, one 50 minute recitation)
Prerequisites: 21-112 or 21-120

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-257 Models and Methods for Optimization

Intermittent: 9 units
Introduces basic methods of operations research and is intended primarily for Business Administration and Economics majors. Review of linear systems; linear programming, including the simplex algorithm, duality, and sensitivity analysis; the transportation problem; the critical path method; the knapsack problem, traveling salesman problem, and an introduction to set covering models. (Three 50 minute lectures, one 50 minute recitation)
Prerequisites: 21-241 or 21-242 or 21-256 or 18-202 or 06-262 or 21-240

21-259 Calculus in Three Dimensions

All Semesters: 10 units
Vectors, lines, planes, quadratic surfaces, polar, cylindrical and spherical coordinates, partial derivatives, directional derivatives, gradient, divergence, curl, chain rule, maximum-minimum problems, multiple integrals, parametric surfaces and curves, line integrals, surface integrals, Green-Gauss theorems. (Three 50 minute lectures, two 50 minute recitations)
Prerequisite: 21-122

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-260 Differential Equations

All Semesters: 9 units
Ordinary differential equations: first and second order equations, applications, Laplace transforms; partial differential equations: partial derivatives, separation of variables, Fourier series; systems of ordinary differential equations; applications. 21-259 or 21-268 or 21-269 are recommended. (Three 50 minute lectures, one 50 minute recitation)
Prerequisite: 21-122

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-261 Introduction to Ordinary Differential Equations

Spring: 10 units
A first course in ordinary differential equations intended primarily for math majors and for those students interested in a more conceptual treatment of the subject. One of the goals of this course is to prepare students for upper level courses on differential equations, mathematical analysis and applied mathematics. Students will be required to write rigorous arguments. Topics to be covered: Ordinary differential equations: first and second order equations, applications, Laplace transform, systems of linear ordinary differential equations; systems of nonlinear ordinary differential equations, equilibria and stability, applications. Corequisites: (21-268 or 21-269 or 21-259) and (21-241 or 21-242). (Three 50 minute lectures, one 50 minute recitation)
Prerequisite: 21-122

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-266 Vector Calculus for Computer Scientists

Spring: 10 units
This course is an introduction to vector calculus making use of techniques from linear algebra. Topics covered include scalar-valued and vector-valued functions, conic sections and quadric surfaces, new coordinate systems, partial derivatives, tangent planes, the Jacobian matrix, the chain rule, gradient, divergence, curl, the Hessian matrix, linear and quadratic approximation, local and global extrema, Lagrange multipliers, multiple integration, parametrised curves, line integrals, conservative vector fields, parametrised surfaces, surface integrals, Green's theorem, Stokes's theorem and Gauss's theorem. (Three 50 minute lectures, one 50 minute recitation)
Prerequisites: 21-122 and (21-242 or 21-241 Min. grade C)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-268 Multidimensional Calculus

Spring: 11 units
A serious introduction to multidimensional calculus that makes use of matrices and linear transformation. Results will be stated carefully and rigorously. Students will be expected to write some proofs; however, some of the deeper results will be presented without proofs. Topics to be covered include: functions of several variables, regions and domains, limits and continuity, partial derivatives, linearization and Jacobian matrices, chain rules, inverse and implicit functions, geometric applications, higher derivatives, Taylor's theorem, optimization, vector fields, multiple integrals and change of variables, Leibnitz's rule, line integrals, Green's theorem, path independence and connectedness, conservative vector fields, surfaces and orientability, surface integrals, divergence theorem and Stokes's theorem. (Three 50 minute lectures, two 50 minute recitations)
Prerequisites: 21-122 and (21-241 or 21-242) and (15-151 or 21-128 or 21-127)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-269 Vector Analysis

Spring: 12 units
A component of the honors program, 21-269 is a more demanding version of 21-268 of greater scope, with greater emphasis placed on rigorous proofs. Topics to be covered typically include: the real field, sups, infs, and completeness; geometry and topology of metric spaces; limits, continuity, and derivatives of maps between normed spaces; inverse and implicit function theorems, higher derivatives, Taylor's theorem, extremal calculus, and Lagrange multipliers. Integration. Iterated integration and change of variables. (Three 50 minute lectures, two 50 minute recitations)
Prerequisites: 21-242 Min. grade B or 21-241 Min. grade A

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-270 Introduction to Mathematical Finance

Spring: 9 units
This is a first course for those considering majoring or minoring in Computational Finance. The theme of this course is pricing derivative securities by replication. The simplest case of this idea, static hedging, is used to discuss net present value of a non-random cash flow, internal rate of return, and put-call option parity. Pricing by replication is then considered in a one-period random model. Risk-neutral probability measures, the Fundamental Theorems of Asset Pricing, and an introduction to expected utility maximization and mean-variance analysis are presented in this model. Finally, replication is studied in a multi-period binomial model. Within this model, the replicating strategies for European and American options are determined. (Three 50 minute lectures)
Prerequisites: 21-112 or 21-120

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-292 Operations Research I

Spring: 9 units
Operations research offers a scientific approach to decision making, most commonly involving the allocation of scarce resources. This course develops some of the fundamental methods used. Linear programming: the simplex method and its linear algebra foundations, duality, post-optimality and sensitivity analysis; the transportation problem; the critical path method; non-linear programming methods. (Three 50 minute lectures, one 50 minute recitation)
Prerequisites: 21-122 and (21-242 or 21-241 or 21-240) and (21-228 or 15-251)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-295 Putnam Seminar

Fall: 3 units
A problem solving seminar designed to prepare students to participate in the annual William Lowell Putnam Mathematical Competition. Students solve and present their solutions to problems posed. (One 50 minute session)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-300 Basic Logic

Fall: 9 units
Propositional and predicate logic: Syntax, proof theory and semantics up to completeness theorem, Lowenheim Skolem theorems, and applications of the compactness theorem. (Three 50 minute lectures)
Prerequisites: 21-228 or 21-373 or 15-251

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-301 Combinatorics

Fall and Spring: 9 units
A major part of the course concentrates on algebraic methods, which are relevant in the study of error correcting codes, and other areas. Topics covered in depth include permutations and combinations, generating functions, recurrence relations, the principle of inclusion and exclusion, and the Fibonacci sequence and the harmonic series. Additional topics may include existence proofs, partitions, finite calculus, generating combinatorial objects, Polya theory, codes, probabilistic methods. (Three 50 minute lectures)
Prerequisites: 21-122 and (21-228 or 15-251)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-302 Lambda Calculus

Intermittent: 9 units
An introductory course in classical lambda calculus, with an emphasis on syntax. The course will describe many research problems which are suitable topics for senior theses or master's theses. Topics will include: Basic properties of reduction and conversion; Reduction and conversion strategies; Calculability and representation of data types; Elementary theory of Ershov numberings; Bohm's theorem, easy terms, and other exotic combinations; Solvability of functional equations (unification); Combinatorics and bases; Simple and algebraic types; Labelled reduction and intersection types; Extensionality and the omega rule.
Prerequisites: 15-150 or 21-300 or 80-310

21-321 Interactive Theorem Proving

Fall: 9 units
Computational proof assistants now make it possible to work interactively to write mechanically verified definitions, theorems, and proofs. Important theorems have been formalized in this way, and digital libraries are being developed collaboratively by the mathematical community. Formalization of mathematics also serves as a gateway to the use of new technologies for discovery, such as automated reasoning and machine learning. This course will teach you how to formalize mathematics so that you, too, can contribute to the effort. We will explore a logical framework, dependent type theory, which serves as a practical foundation in a number of proof assistants. Finally, as time allows, we will explore ways of automating various aspects of mathematical reasoning. (Three 50 minute lectures)
Prerequisites: 21-128 or 21-127 or 15-151

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-322 Topics in Formal Mathematics

Intermittent: 9 units
Complementary to 21-321 Interactive Theorem Proving, this course is designed to present special topics on the use of formalization and formal methods in mathematics. For example, the course might focus on formalization of a specific branch of mathematics, or on a specific method for automation or computational reasoning.
Prerequisites: (21-128 or 21-127 or 15-151) and (21-259 or 21-266 or 21-268 or 21-269)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-325 Probability

Fall and Spring: 9 units
This course focuses on the understanding of basic concepts in probability theory and illustrates how these concepts can be applied to develop and analyze a variety of models arising in computational biology, finance, engineering and computer science. The firm grounding in the fundamentals is aimed at providing students the flexibility to build and analyze models from diverse applications as well as preparing the interested student for advanced work in these areas. The course will cover core concepts such as probability spaces, random variables, random vectors, multivariate densities, distributions, expectations, sampling and simulation; independence, conditioning, conditional distributions and expectations; limit theorems such as the strong law of large numbers and the central limit theorem; as well as additional topics such as large deviations, random walks and Markov chains, as time permits. (Three 50 minute lectures)
Prerequisites: 21-269 or 21-268 or 21-256 or 21-259

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-329 Set Theory

Spring: 9 units
Set theory was invented about 110 years ago by George Cantor as an instrument to understand infinite objects and to compare different sizes of infinite sets. Since then set theory has come to play an important role in several branches of modern mathematics, and serves as a foundation of mathematics. Contents: Basic properties of natural numbers, countable and uncountable sets, construction of the real numbers, some basic facts about the topology of the real line, cardinal numbers and cardinal arithmetic, the continuum hypothesis, well ordered sets, ordinal numbers and transfinite induction, the axiom of choice, Zorn's lemma. Optional topics if time permits: Infinitary combinatorics, filters and large cardinals, Borel and analytic sets of reals. (Three 50 minute lectures)
Prerequisites: 15-151 or 21-127 or 21-128

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-341 Linear Algebra

Fall and Spring: 9 units
A mathematically rigorous treatment of Linear Algebra over an arbitrary field. Topics studied will include abstract vector spaces, linear transformations, determinants, eigenvalues, eigenvectors, inner products, invariant subspaces, canonical forms, the spectral theorem and the singular value decomposition. 21-373 recommended. (Three 50 minute lectures)
Prerequisites: (21-241 or 21-242) and 21-373

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-344 Numerical Linear Algebra

Spring: 9 units
An introduction to algorithms pertaining to matrices and large linear systems of equations. Direct methods for large sparse problems including graph data structures, maximum matchings, row and column orderings, and pivoting strategies. Iterative methods including Conjugate Gradient and GMRES, with a discussion of preconditioning strategies. Additional topics include: computation of eigenvalues and eigenvectors, condition numbers, the QR and singular value decompositions, least-squares systems. (Three 50 minute lectures)
Prerequisites: 15-112 and (21-242 or 21-241 or 21-240) and (21-269 or 21-259 or 21-268)

21-355 Principles of Real Analysis I

Fall and Spring: 9 units
This course provides a rigorous and proof-based treatment of functions of one real variable. The course presumes some mathematical sophistication including the ability to recognize, read, and write proofs. Topics include: The Real Number System: Field and order axioms, sups and infs, completeness. Real Sequences. Bolzano-Weierstrass theorem. Topology of the Real Line: Open sets, closed sets, compactness, Heine-Borel Theorem. Continuity: extreme and intermediate value theorems, uniform continuity. Differentiation: chain rule, local extrema, mean-value theorem, L'Hospital's rule, Taylor's theorem. Riemann integration: sufficient conditions for integrability, fundamental theorems of calculus. (Three 50 minute lectures)
Prerequisites: (15-151 or 21-128 or 21-127) and 21-122

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-356 Principles of Real Analysis II

Fall and Spring: 9 units
This course provides a rigorous and proof-based treatment of functions of several real variables. The course presumes some mathematical sophistication including the ability to recognize, read, and write proofs. Topics include: Metric spaces. Differential calculus in Euclidean spaces: continuity, differentiability, partial derivatives, gradients, differentiation rules, implicit and inverse function theorems. Multiple integrals. Integration on curves and hypersurfaces: arclength, and generalized area. The divergence theorem and the 3D Stokes theorem. Regarding prerequisites, 21-268 or 21-269 are strongly recommended rather than 21-259. (Three 50 minute lectures)
Prerequisites: (21-269 or 21-268 or 21-259) and (21-242 or 21-241) and (21-355 or 21-455)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-360 Differential Geometry of Curves and Surfaces

Intermittent: 9 units
The course is a rigorous introduction to the differential and integral calculus of curves and surfaces. Topics to be covered include: Parameterized and regular curves Frenet equations canonical coordinate system, local canonical forms, global properties of plane curves Regular surfaces, differential functions on surfaces, the tangent plane and differential of a map, orientation of surfaces, characterization of compact orientable surfaces, classification of compact surfaces The geometry of the Gauss map, isometries and conformal maps, parallel transport, geodesics, the Gauss-Bonnet theorem and applications. More topics may be covered, as time allows. Students should be prepared to write proofs and perform computations. 21-356 or 21-236 are recommended. (Three 50 minute lectures)
Prerequisites: 21-269 or 21-268

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-365 Projects in Applied Mathematics

Intermittent: 9 units
This course provides students with an opportunity to solve problems posed by area companies. It is also designed to provide experience working as part of a team to solve problems for a client. The background needed might include linear programming, simulation, data analysis, scheduling, numerical techniques, etc.

21-366 Topics in Applied Mathematics

Intermittent: 9 units
This course affords students with the opportunity to study topics which are in the area of expertise of the instructor. This course may taken more than once if content is sufficiently different. Course prerequisites will depend on the content of the course. Please see the course URL for semester-specific topics. (Three 50 minute lectures)
Prerequisite: 21-228

Course Website: https://www.cmu.edu/math/courses/special-topics.html

21-369 Numerical Methods

Fall and Spring: 12 units
This course provides an introduction to the use of computers to solve scientific problems. Methods for the computational solution of linear algebra systems, nonlinear equations, the interpolation and approximation of functions, differentiation and integration, and ordinary differential equations. Analysis of roundoff and discretization errors and programming techniques. 21-268 or 21-269 are recommended prerequisites, rather than 21-259. (Three 50 minute lectures, one 50 minute recitation)
Prerequisites: (15-110 or 15-112) and (21-269 or 21-268 or 21-259) and (21-242 or 21-241 or 21-240 or 33-232) and (21-261 or 21-630 or 21-260 or 33-231)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-370 Discrete Time Finance

Fall: 9 units
This course introduces the Black-Scholes option pricing formula, shows how the binomial model provides a discretization of this formula, and uses this connection to fit the binomial model to data. It then sets the stage for Continuous-Time Finance by discussing in the binomial model the mathematical technology of filtrations, martingales, Markov processes and risk-neutral measures. Additional topics are American options, expected utility maximization, the Fundamental Theorems of Asset Pricing in a multi-period setting, and term structure modeling, including the Heath-Jarrow-Morton model. Students in 21-370 are expected to read and write proofs. Acceptable corequisites include 21-325 or 36-225 (Three 50 minute lectures)
Prerequisites: (21-270 or 70-492) and (21-269 or 21-259 or 21-256 or 21-268)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-371 Functions of a Complex Variable

Fall: 9 units
This course provides an introduction to one of the basic topics of both pure and applied mathematics and is suitable for those with both practical and theoretical interests. Algebra and geometry of complex numbers; complex differentiation and integration. Cauchy's theorem and applications; conformal mapping; applications. 21-268 or 21-269 are recommended prerequisites, rather than 21-259. (Three 50 minute lectures)
Prerequisites: 21-355 or 21-455 or 21-235

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-373 Algebraic Structures

Fall and Spring: 9 units
Groups: Homomorphisms. Subgroups, cosets, Lagrange's theorem. Conjugation. Normal subgroups, quotient groups, first isomorphism theorem. Group actions, Cauchy's Theorem. Dihedral and alternating groups. The second and third isomorphism theorems. Rings: Subrings, ideals, quotient rings, first isomorphism theorem. Polynomial rings. Prime and maximal ideals, prime and irreducible elements. PIDs and UFDs. Noetherian domains. Gauss' lemma. Eisenstein criterion. Fields: Field of fractions of an integral domain. Finite fields. Applications to coding theory, cryptography, number theory. (Three 50 minute lectures)
Prerequisites: (15-151 or 21-128 or 21-127) and (21-242 or 21-241)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-374 Field Theory

Spring: 9 units
The purpose of this course is to provide a successor to Algebraic Structures, with an emphasis on applications of groups and rings within algebra to some major classical problems. These include constructions with a ruler and compass, and the solvability or unsolvability of equations by radicals. It also offers an opportunity to see group theory and basic ring theory "in action", and introduces several powerful number theoretic techniques. The basic ideas and methods required to study finite fields will also be introduced. These ideas have recently been applied in a number of areas of theoretical computer science including primality testing and cryptography. (Three 50 minute lectures)
Prerequisite: 21-373

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-375 Topics in Algebra

Intermittent: 9 units
Typical of courses that might be offered from time to time are elliptic curves, commutative algebra, and theory of Boolean functions. (Three 50 minute lectures)
Prerequisite: 21-373

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-377 Monte Carlo Simulation for Finance

Intermittent: 9 units
first course in Monte Carlo simulation, with applications to Mathematical Finance. Students will put into practice many of the theoretical ideas introduced in Continuous Time Finance. Topics to be covered: random variable/stochastic process generation; options pricing; variance reduction; Markov chain Monte Carlo Methods.
Prerequisites: 21-420 or 21-325 Min. grade B

21-378 Mathematics of Fixed Income Markets

Fall: 9 units
A first course in fixed income. Students will be introduced to the most common securities traded in fixed income markets and the valuation methods used to price them. Topics covered include discount factors; interest rates basics; pricing of coupon bonds; identifying the yield to maturity, as well as bond sensitivities to interest rates; term structure modeling; forward and swap rates; fixed income derivatives (including mortgage backed securities) and their valuation through backwards induction; fixed income indexes and return attribution. For a co-requisite, 36-225 can be accepted as an alternative for 21-325. (Three 50 minute lectures)
Prerequisite: 21-270 Min. grade B

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-380 Introduction to Mathematical Modeling

Intermittent: 9 units
This course shall examine mathematical models, which may be used to describe natural phenomena. Examples, which have been studied include: continuum description of highway traffic, discrete velocity models of a monotonic gas, chemotactic behavior in biological systems, European options pricing, and cellular-automata. Systems such as the first four are described by partial differential equations; the last involves discrete-time and discrete-phase dynamical systems, which have been used to successfully represent both physical and biological systems. The course will develop these models and then examine the behavior of the underlying systems, both analytically and numerically. The mathematical tools required will be developed in the course. (Three 50 minute lectures)
Prerequisites: (21-241 or 21-242) and (21-261 or 21-260)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-387 Monte Carlo Methods and Applications

Intermittent: 9 units
The Monte Carlo method uses random sampling to solve computational problems that would otherwise be intractable, and enables computers to model complex systems in nature that are otherwise too difficult to simulate. This course provides a first introduction to Monte Carlo methods from complementary theoretical and applied points of view, and will include implementation of practical algorithms. Topics include random number generation, sampling, Markov chains, Monte Carlo integration, stochastic processes, and applications in computational science. Students need a basic background in probability, multivariable calculus, and some coding experience in any language. (Two 80 minute lectures)
Prerequisites: (21-259 Min. grade C or 21-269 Min. grade C or 21-266 Min. grade C or 21-268 Min. grade C) and (36-235 Min. grade C or 15-259 Min. grade C or 36-225 Min. grade C or 36-219 Min. grade C or 21-325 Min. grade C or 18-465 Min. grade C or 36-218 Min. grade C)

Course Website: http://www.cs.cmu.edu/~kmcrane/random/

21-393 Operations Research II

Fall: 9 units
Building on an understanding of Linear Programming developed in 21-292 Operations Research I, this course introduces more advanced topics. Integer programming, including cutting planes and branch and bound. Dynamic programming. An introduction to Combinatorial Optimization including optimal spanning trees, shortest paths, the assignment problem and max-flow/min-cut. The traveling salesman problem and NP-completeness. An important goal of this course is for the student to gain experience with the process of working in a group to apply operations research methods to solve a problem. A portion of the course is devoted to a group project based upon case studies and the methods presented. 36-410 recommended. (Three 50 minute lectures)
Prerequisites: (15-251 or 21-228) and 21-292

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-400 Intermediate Logic

Intermittent: 9 units
The course builds on the proof theory and model theory of first-order logic covered in 21-300. These are applied in 21-400 to Peano Arithmetic and its standard model, the natural numbers. The main results are the incompleteness, undefinability and undecidability theorems of Godel, Tarski, Church and others. Leading up to these, it is explained how logic is formalized within arithmetic, how this leads to the phenomenon of self-reference, and what it means for the axioms of a theory to be computably enumerable. Related aspects of computability theory are included to the extent that time permits. (Three 50 minute lectures)
Prerequisite: 21-300

21-410 Research Topics in Mathematical Sciences

Intermittent: 9 units
This course affords undergraduates to pursue elementary research topics in the area of expertise of the instructor. The prerequisites will depend on the content of the course. (Three 50 minute lectures)

Course Website: https://www.cmu.edu/math/courses/special-topics.html

21-420 Continuous-Time Finance

Spring: 9 units
This course begins with Brownian motion, stochastic integration,and Ito's formula from stochastic calculus. This theory is used to develop the Black-Scholes option pricing formula and the Black-Scholes partial differential equation. Additional topics may include models of credit risk, simulation, and expected utility maximization. (Three 50 minute lectures)
Prerequisites: (21-260 or 18-202) and 21-370 and (36-225 or 36-217 or 15-259 or 36-218 or 21-325)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-435 Applied Harmonic Analysis

Intermittent: 9 units
This course serves as a broad introduction to harmonic analysis and its applications, particularly in 1-dimensional signal processing and in image processing, for undergraduate students in mathematics, engineering, and the applied sciences. Topics include: Discrete Fourier transform and fast Fourier transform; Fourier series and the Fourier transform; Hilbert spaces and applications; Shannon sampling theorem, bandlimited functions, uncertainty principle; Wavelets and multi-resolution analysis; Applications in image processing. (Three 50 minute lectures)
Prerequisites: (21-355 or 21-235 or 21-455) and (21-242 or 21-241)

21-441 Number Theory

Fall: 9 units
Number theory deals with the integers, the most basic structures of mathematics. It is one of the most ancient, beautiful, and well-studied branches of mathematics, and has recently found surprising new applications in communications and cryptography. Course contents: Structure of the integers, greatest common divisors, prime factorization. Modular arithmetic, Fermat's Theorem, Chinese Remainder Theorem. Number theoretic functions, e.g. Euler's function, Mobius functions, and identities. Diophantine equations, Pell's Equation, continued fractions. Modular polynomial equations, quadratic reciprocity. (Three 50 minute lectures)
Prerequisites: (21-242 or 21-241) and 21-373

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-455 Intermediate Real Analysis I

Fall: 12 units
This course provides a rigorous and proof-based treatment of the general theory of functions on metric spaces. The course serves as a more advanced version of 21-355 Principles of Real Analysis and is primarily intended for students who have taken 21-269. Topics include: Metric spaces: Completeness, density, separability, compactness, connectedness. Baire theorem and applications. Contraction maps: fixed points, applications, inverse, and implicit function theorems. Spaces of functions: uniform and pointwise convergence, Stone-Weierstrass theorem, Arzela-Ascoli theorem. (Three 50 minute lectures, one 50 minute recitation)
Prerequisites: 21-269 or 21-268

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-456 Intermediate Real Analysis II

Spring: 10 units
This course provides a rigorous and proof-based treatment of the general theory of functions on Euclidean spaces. The course serves as a more advanced version of 21-356 Principles of Real Analysis II and is primarily intended for students who have taken 21-455. Topics include: Lebesgue integration in Euclidean spaces: Lebesgue measure and Lebesgue integration, convergence and integration theorems, Fubini's theorem, change of variables. Curves, arclength, curve integrals. Submanifolds of Euclidean space: applications of the inverse and implicit function theorems, tangent space, normal space, orientation, integration on manifolds. Theorems of vector calculus: divergence theorem, Stokes theorem in 3D. (Three 50 minute lectures, one 50 minute recitations)
Prerequisites: 21-455 or 21-355 or 21-235

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-465 Topology

Intermittent: 9 units
Metric spaces. Topological spaces. Separation axioms. Open, closed and compact sets. Continuous functions. Product spaces, subspaces, quotient spaces. Connectedness and path-connectedness. Homotopy. Fundamental group of a pointed space. Simply connected spaces. Winding number, the fundamental group of the circle. Functorial property of the fundamental group. Brouwer fixed point theorem. Covering spaces. van Kampen's theorem. 2-manifolds. Triangulations. Euler characteristic. Surgery, classification of compact 2-manifolds. (Three 50 minute lectures)
Prerequisites: (21-455 or 21-355 or 21-235) and 21-373

21-469 Computational Introduction to Partial Differential Equations

Intermittent: 12 units
A Partial Differential Equation (PDE for short) is a differential equation involving derivatives with respect to more than one variable. These arise in numerous applications from various disciplines. Most PDEs do not have explicit solutions, and hence computational methods are essential for understanding the underlying phenomena. This course will serve as a first introduction to PDEs and their numerical approximation, and will focus on a variety of mathematical models. It will cover both analytical methods, numerical methods (e.g. finite differences) and the use of a computer to approximate and visualize solutions. The mathematical ideas behind phenomena observed in nature will be studied at the theoretical level and in numerical simulations (e.g. speed of wave propagation, and/or shocks in traffic flow). Topics will include: Derivation of PDEs from physical principles, analytical and computational tools for the transport equation and the Poisson equation, Fourier analysis, analytical and numerical techniques for the solution of parabolic equations and if time permits, the wave equation. (Three 50 minute lectures, one 80 minute recitation)
Prerequisites: (21-240 or 21-242 or 21-241) and (21-269 or 21-268 or 21-259) and (21-261 or 33-231 or 21-260 or 21-630) and (15-112 or 15-110)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-470 Selected Topics in Analysis

Intermittent: 9 units
Typical of courses, which are offered from time to time are finite difference equations, calculus of variations, and applied control theory. The prerequisites will depend on the content of the course. (Three 50 minute lectures)
Prerequisites: 21-260 and 21-241 and 21-259

21-476 Introduction to Dynamical Systems

Intermittent: 9 units
This course is an introduction to differentiable dynamical systems. The material includes basic properties of dynamical systems, including the existence and uniqueness theory, continuation, singular points, orbits, and their classification. The Poincare'-Bendixson theorem and typical applications, like Lienard equations and Lotka-Volterra are also covered. An introduction to chaos as time permits. (Three 50 minute lectures)
Prerequisites: (21-241 or 21-242) and 21-261

21-484 Graph Theory

Spring: 9 units
Graph theory uses basic concepts to approach a diversity of problems and nontrivial applications in operations research, computer science and other disciplines. It is one of the very few mathematical areas where one is always close to interesting unsolved problems. Topics include graphs and subgraphs, trees, connectivity, Euler tours and Hamilton cycles, matchings, graph colorings, planar graphs and Euler's Formula, directed graphs, network flows, counting arguments, and graph algorithms. (Three 50 minute lectures)
Prerequisites: (15-251 or 21-228) and (21-241 or 21-242)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-590 Practicum in Mathematical Sciences

All Semesters: 3 units
Students in this course gain experience with the application of mathematical models to business and/or industrial problems during an internship. The internship is set up by the student in consultation with a faculty member. The students must also have a mentor at the firm providing the internship, who together with the faculty member develops a description of the goals of the internship. The internship must include the opportunity to learn about problems which have mathematical content. Tuition is charged for this course.

21-599 Undergraduate Reading and Research

Fall and Spring
Individual reading courses or projects in mathematics and its applications. Prerequisites and units to be negotiated with individual instructors.

21-602 Introduction to Set Theory I

Fall: 12 units
The axioms of ZFC, ordinal arithmetic, cardinal arithmetic including Konig's lemma, class length induction and recursion, the rank hierarchy, the Mostowski collapse theorem, the H-hierarchy, the Delta_1 absoluteness theorem, the absoluteness of wellfoundedness, the reflection theorem for hierarchies of sets, ordinal definability, the model HOD, relative consistency, Goedel's theorem that HOD is a model of ZFC, constructibility, Goedel's theorem that L is a model of ZFC + GCH, the Borel and Projective hierarchies and their effective versions, Suslin representations for Sigma^1_1, Pi^1_1 and Sigma^1_2, sets of reals, Shoenfield's absoluteness theorem, the complexity of the set of constructible reals, the combinatorics of club and stationary sets (including the diagonal intersection, the normality of the club filter, Fodor's lemma and its applications), Solovay's splitting theorem, model theoretic techniques commonly applied in set theory (e.g., elementary substructures, chains of models and ultrapowers), club and stationary subsets of [X]^/omega and their combinatorics, Jensen's diamond principles and his proofs that they hold in L, Gregory's theorem and generalizations, constructions of various kinds of uncountable trees (including Aronszajn, special, Suslin, Kurepa), Jensen's square principles and elementary applications, the basic theory of large cardinals (including inaccesssible, Mahlo, weakly compact and measurable cardinals), Scott's theorem that there are no measurable cardinals in L, Kunen's theorem that the only elementary embedding from V to V is the identity. Optional topic: SCH and Silver's theorem. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-455 Min. grade B or 21-235 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B) and 21-329 Min. grade B and 21-300 Min. grade B

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-603 Model Theory I

Intermittent: 12 units
Similarity types, structures; downward Lowenheim Skolem theorem; construction of models from constants, Henkin's omitting types theory, prime models; elementary chains of models, basic two cardinal theorems, saturated models, basic results on countable models including Ryll-Nardzewski's theorem; indiscernible sequences, Ehrenfeucht-Mostowski models; introduction to stability, rank functions, primary models, and a proof of Morley's catagoricity theorem; basic facts about infinitary languages, computation of Hanf-Morley numbers. (Three 50 minute lectures)
Prerequisites: (21-455 Min. grade B or 21-235 Min. grade B or 21-355 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-604 Introduction to Recursion Theory

Intermittent: 12 units
Models of computation, computable functions, solvable and unsolvable problems, reducibilities among problems, recursive and recursively enumerable sets, the recursion theorem, Post's problem and the Friedberg-Muchnik theorem, general degrees and r.e. degrees, the arithmetical hierarchy, the hyper-arithmetical hierarchy, the analytical hierarchy, higher type recursion. (Three 50 minute lectures)
Prerequisites: (21-455 Min. grade B or 21-235 Min. grade B or 21-355 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-610 Algebra I

Intermittent: 12 units
The structure of finitely generated abelian groups, the Sylow theorems, nilpotent and solvable groups, simplicity of alternating and projective special linear groups, free groups, the Neilsen-Schreier theorem. Vector spaces over division rings, field extensions, the fundamental Galois correspondence, algebraic closure. The Jacobson radical and the structure of semisimple rings. Time permitting, one of the following topics will be included: Wedderburn's theorem on finite division rings, Frobenius' Theorem. Prerequisite: Familiarity with the content of an undergraduate course on groups and rings. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-235 Min. grade B or 21-455 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-620 Real Analysis

Fall: 6 units
A review of one-dimensional, undergraduate analysis, including a rigorous treatment of the following topics in the context of real numbers: sequences, compactness, continuity, differentiation, Riemann integration. (Mini-course. Normally taken with 21-621.) (Three 50 minute lectures)
Prerequisites: (21-455 Min. grade B or 21-235 Min. grade B or 21-355 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-621 Introduction to Lebesgue Integration

Fall: 6 units
Construction of Lebesgue measure and the Lebesgue integral on the real line. Fatou's Lemma, the monotone convergence theorem, the dominated convergence theorem. (Mini-course. Normally taken with 21-620.) (Three 50 minute lectures)
Prerequisites: (21-235 Min. grade B or 21-355 Min. grade B or 21-455 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B) and 21-620 Min. grade B

21-623 Complex Analysis

Intermittent: 12 units
The complex plane, holomorphic functions, power series, complex integration, and Cauchy's Theorem. Calculus of residues. Additional topics may include conformal mappings and the application of complex transforms to differential equations. (Three 50 minute lectures)
Prerequisites: (21-236 Min. grade B or 21-356 Min. grade B or 21-456 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-624 Descriptive Set Theory

Intermittent: 12 units
The central theme of the course is the study of "definable" subsets of Polish spaces (a class of topological spaces containing many spaces studied across mathematics). In some sense, restricting attention to definable sets makes life nicer; for instance, (properly interpreted) the continuum hypothesis becomes true! And weird Banach-Tarski stuff doesn't happen. However, it comes at a price: equivalence relations can have more classes than elements, and acyclic graphs can become hard to properly color. For the first portion of the course we will work through the basic theory of Borel and analytic subsets of Polish spaces, highlighting their interaction with measure and Baire category. We will then focus on the modern theory of equivalence relations, paying special attention to orbit equivalence relations of group actions and connectedness relations of graphs. Along the way we will establish several classical dichotomy theorems. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-455 Min. grade B or 21-235 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B) and 21-329 Min. grade B

21-630 Ordinary Differential Equations

Intermittent: 12 units
Basic concepts covered are existence and uniqueness of solutions, continuation of solutions, continuous dependence, and stability. For autonomous systems, topics included are: orbits, limit sets, Liapunov's direct method, and Poincar-Bendixson theory. For linear systems, topics included are: fundamental solutions, variation of constants, stability, matrix exponential solutions, and saddle points. Time permitting, one or more of the following topics will be covered: differential inequalities, boundary-value problems and Sturm-Liouville theory, Floquet theory. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-235 Min. grade B or 21-455 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-632 Introduction to Differential Equations

Fall: 12 units
This course serves as a broad introduction to Ordinary and Partial Differential Equations for beginning graduate students and advanced undergraduate students in mathematics, engineering, and the applied sciences. Mathematical sophistication in real analysis at the level of 21-355/356 is assumed. Topics include: essentials of Ordinary Differential Equations, origins of Partial Differential Equations, the study of model problems including the Poisson and Laplace equations, the heat equation, the transport equation, and the wave equation. (Three 50 minute lectures)
Prerequisites: (21-356 Min. grade B or 21-236 Min. grade B or 21-456 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B)

21-640 Introduction to Functional Analysis

Spring: 12 units
Linear spaces: Hilbert spaces, Banach spaces, topological vector spaces. Hilbert spaces: geometry, projections, Riesz Representation Theorem, bilinear and quadratic forms, orthonormal sets and Fourier series. Banach spaces: continuity of linear mappings, Hahn-Banach Theorem, uniform boundedness, open-mapping theorem. Closed operators, closed graph theorem. Dual spaces: weak and weak-star topologies (Banach-Alaoglu Theorem), reflexivity. Space of bounded continuous functions and its dual. Linear operators and adjoints: basic properties, null spaces and ranges. Compact operators. Sequences of bounded linear operators: weak, strong and uniform convergence. Introduction to spectral theory: Notions of spectrum and resolvent set of bounded operators, spectral theory of compact operators. Time permitting: Fredholm Alternative. Time permitting: Stone-Weierstrass Theorem. (Three 50 minute lectures)
Prerequisites: (21-236 Min. grade B or 21-651 Min. grade B) and (21-720 Min. grade B or 21-236 Min. grade B)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-651 General Topology

Fall: 12 units
Metric spaces: continuity, compactness, Arzela-Ascoli Theorem, completeness and completion, Baire Category Theorem. General topological spaces: bases and subbases, products, quotients, subspaces, continuity, topologies generated by sets of functions, homeomorphisms. Convergence: nets, filters, and the inadequacy of sequences. Separation: Hausdorff spaces, regular spaces, completely regular spaces, normal spaces, Urysohn's Lemma, Tietze's Extension Theorem. Connectedness. Countability conditions: first and second countability, separability, Lindelof property. Compactness: Tychonoff's Theorem, local compactness, one-point compactification. (Three 50 minute lectures)
Prerequisites: (21-455 Min. grade B or 21-235 Min. grade B or 21-355 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-660 Introduction to Numerical Analysis I

Spring: 12 units
Finite precision arithmetic, interpolation, spline approximation, numerical integration, numerical solution of linear and nonlinear systems of equations, optimization in finite dimensional spaces. (Three 50 minute lectures)
Prerequisites: (21-456 Min. grade B or 21-356 Min. grade B or 21-236 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-670 Linear Algebra for Data Science

Fall: 6 units
This course is designed to present and discuss those aspects of Linear Algebra that are most important in Data Analytics. The emphasis will be on developing intuition and understanding how to use linear algebra, rather than on proofs. (Three 50 minute lectures)
Prerequisites: (21-455 Min. grade B or 21-355 Min. grade B or 21-235 Min. grade B) and 21-373 Min. grade B

21-671 Computational Linear Algebra

Fall: 12 units
This is a survey of methods in computational linear algebra. Topics covered in this course focus around algorithms for solving (dense or large and sparse) linear systems. Regularization and underdetermined systems will be discussed in detail. Rather than assuming prior knowledge in numerical analysis or matrix theory, we will introduce standard methods or results when needed. In this way, much of the material is self-contained. Theoretical and experimental results will be covered accordingly, with an emphasis on cost, stability, and convergence. (Three 50 minute lectures)
Prerequisites: (21-240 or 21-242 or 21-241) and (21-268 or 21-269 or 21-259)

21-681 Stochastic Calculus in Finance

Intermittent: 6 units
This is a graduate-level introduction to continuous-time equilibrium asset pricing models. Using tools from Ito calculus, the first part of the course covers the benchmark case of complete, frictionless markets, for which a fairly general theory and a number of solvable examples have been developed. The second part of the course then provides an overview of cutting-edge research on extensions of the baseline model that account for "flaws and frictions" such as heterogeneous beliefs, trading costs, or asymmetric information. In the third part of the course, students will present a related research paper, chosen together with the instructor in accordance with their background and research interests. (One 80 minute lecture)
Prerequisites: (21-355 Min. grade B or 21-235 Min. grade B or 21-455 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-690 Methods of Optimization

Intermittent: 12 units
An introduction to the theory and algorithms of linear and nonlinear programming with an emphasis on modern computational considerations. The simplex method and its variants, duality theory and sensitivity analysis. Large-scale linear programming. Optimality conditions for unconstrained nonlinear optimization. Newton's method, line searches, trust regions and convergence rates. Constrained problems, feasible-point methods, penalty and barrier methods, interior-point methods. (Three 50 minute lectures)
Prerequisites: (21-356 Min. grade B or 21-236 Min. grade B or 21-456 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

Course Website: https://www.cmu.edu/math/undergrad/math-course-information.html

21-701 Discrete Mathematics

Fall: 12 units
Combinatorial analysis, graph theory with applications to problems in computational complexity, networks, and other areas. (Three 50 minute lectures)
Prerequisites: (21-455 Min. grade B or 21-235 Min. grade B or 21-355 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-702 Set Theory II

Spring: 12 units
This course is a sequel to 21-602 Set Theory. The main goal is to prove Solovay's theorem that Con(ZFC + an inaccessible cardinal) implies Con(ZF + DC + every set of reals is Lebesgue measurable). Topics covered include absoluteness theorems, Borel codes, the Levy collapse, product forcing, relative constructibility, and the basics of iterated forcing up to the consistency of Martin's Axiom. (Three 50 minute lectures)
Prerequisites: (21-235 Min. grade B or 21-355 Min. grade B or 21-455 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B) and 21-602 Min. grade B

21-703 Model Theory II

Spring: 12 units
The course concentrates in what is considered "main stream model theory" with is Shelah's classification theory (known also as Stability). Among the topics to be presented are stability, superstability, the theory of various notions of primeness, rank functions, forking calculus, the stability spectrum theorem, finite equivalence relations theorem, stable groups (up to and including the Macintyre-Cherlin-Shelah theorem on super-stable fields), and some elementary geometric model theory. If time permits also: basic facts about infinitary languages, computation of Hanf-Morley numbers; some of the Ax-Kochen-Ershov theory of model theory for fields with valuations (will apply this to solve Artin's conjecture). (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-235 Min. grade B or 21-455 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B) and 21-603 Min. grade B

Course Website: http://www.math.cmu.edu/~rami/mt2.11.desc.html

21-720 Measure and Integration

Fall: 12 units
The Lebesgue integral, absolute continuity, signed measures and the Radon-Nikodym Theorem, Lp spaces and the Riesz Representation Theorem, product measures and Fubini's Theorem. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-455 Min. grade B or 21-235 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B)

21-721 Probability

Spring: 12 units
Probability spaces, random variables, expectation, independence, Borel-Cantelli lemmas. Kernels and product spaces, existence of probability measures on infinite product spaces, Kolmogorov's zero-one law. Weak and strong laws of large numbers, ergodic theorems, stationary sequences. Conditional expectation: characterization, construction and properties. Relation to kernels, conditional distribution, density. Filtration, adapted and predictable processes, martingales, stopping times, upcrossing inequality and martingale convergence theorems, backward martingales, optional stopping, maximal inequalities. Various applications of martingales: branching processes, Polya's urn, generalized Borel-Cantelli, Levy's 0-1 law, martingale method, strong law of large numbers, etc. Weak convergence of probability measures, characteristic functions of random variables, weak convergence in terms of characteristic functions. Central limit theorem, Poisson convergence, Poisson process. Large deviations, rate functions, Cramer's Theorem. (Three 50 minute lectures)
Prerequisites: (21-235 Min. grade B or 21-355 Min. grade B or 21-455 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B) and 21-721 Min. grade B

21-723 Advanced Real Analysis

Spring: 12 units
This course is a sequel to 21-720 Measure and Integration. It is meant to introduce students to a number of important advanced topics in analysis.Topics include: distributions, Fourier series and transform, Sobolev spaces, Bochner integration, basics of interpolation theory, integral transforms. (Three 50 minute lectures)
Prerequisites: (21-356 Min. grade B or 21-236 Min. grade B or 21-456 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B) and 21-720 Min. grade B

21-724 Sobolev Spaces

Intermittent: 12 units
Weak derivatives, Sobolev spaces of integer order, embedding theorems, interpolation inequalities, traces. (Three 50 minute lectures)
Prerequisites: (21-356 Min. grade B or 21-456 Min. grade B or 21-236 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B) and 21-720 Min. grade B

21-732 Partial Differential Equations I

Fall: 12 units
An introduction to the modern theory of partial differential equations. Including functional analytic techniques. Topics vary slightly from year to year, but generally include existence, uniqueness and regularity for linear elliptic boundary value problems and an introduction to the theory of evolution equations. (Three 50 minute lectures)
Prerequisites: 21-723 Min. grade B and 21-632 Min. grade B and 21-640 Min. grade B and (21-237 Min. grade B or 21-373 Min. grade B)

21-737 Probabilistic Combinatorics

Intermittent: 12 units
This course covers the probabilistic method for combinatorics in detail and introduces randomized algorithms and the theory of random graphs. Methods covered include the second moment method, the R and #246;dl nibble, the Lov and #225;sz local lemma, correlation inequalities, martingale's and tight concentration, Janson's inequality, branching processes, coupling and the differential equations method for discrete random processes. Objects studied include the configuration model for random regular graphs, Markov chains, the phase transition in the Erd and #246;s-R and #233;nyi random graph, and the Barab and #225;si-Albert preferential attachment model. (Three 50 minute lectures)
Prerequisites: 21-301 and (21-325 or 15-259 or 36-225 or 36-218)

21-738 Extremal Combinatorics

Intermittent: 12 units
Classical problems and results in extremal combinatorics including the Tur and #225;n and Zarankiewicz problems, the Erdos-Stone theorem and the Erdos-Simonovits stability theorem. Extremal set theory including the Erdos-Rado sunflower lemma and variations, VC-dimension, and Kneser's conjecture. The Szemeredi regularity lemma. Algebraic methods including finite field constructions and eigenvalues and expansion properties of graphs. Shannon capacity of graphs. Chromatic number of Rn and Borsuk's conjecture. Graph decomposition including Graham-Pollack and Baranyai's theorem. (Three 50 minute lectures)
Prerequisites: (21-235 Min. grade B or 21-455 Min. grade B or 21-355 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-742 Calculus Of Variations

Intermittent: 12 units
Classical fixed endpoint examples. Fixed endpoint problems in classes of absolutely continuous functions: existence via lower semicontinuity. Tonelli's existence theorem. Euler-Lagrange and DuBois Reymond equations, transversality conditions, Weierstrass field theory, Hamilton-Jacobi theory. Problems with constraints. (Three 50 minute lectures)
Prerequisites: (21-455 Min. grade B or 21-235 Min. grade B or 21-355 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B) and 21-723 Min. grade B

21-752 Algebraic Topology

Intermittent: 12 units
Topology is a less rigid variant of geometry that studies shapes of spaces. Algebraic topology associates algebraic invariants, such as groups or rings, to such spaces. This is achieved by building a space from simpler ones or by algebraically keeping track of how to map a simple space into a given space. This course will cover the fundamental group and covering spaces, homology theories, and the cohomology ring of a space (time permitting). (Three 50 minute lectures)
Prerequisites: 21-651 Min. grade B and (21-455 Min. grade B or 21-355 Min. grade B or 21-235 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-759 Differential Geometry

Intermittent: 12 units
Manifolds in Euclidean spaces, curves and surfaces, principal curvatures, geodesics. Surfaces with constant mean curvature, minimal surfaces. Abstract differentiable manifolds, tangent spaces, vector bundles, affine connections, parallelisms, covariant gradients, Cartan torsion, Riemann curvature. Riemannian geometry, Lie groups. Familiarity with analysis in finite dimensional spaces will be assumed. (Three 50 minute lectures)
Prerequisites: (21-236 Min. grade B or 21-356 Min. grade B or 21-456 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B)

21-762 Finite Element Methods

Intermittent: 12 units
Finite element methods for elliptic boundary value problems. Analysis of errors, approximation by finite element spaces. Efficient implementation of finite element spaces. Efficient implementation of finite element algorithms, finite element methods for parabolic and eigenvalue problems, effects of curved boundaries. Numerical quadrature, non-conforming methods. (Three 50 minute lectures)
Prerequisites: (21-456 Min. grade B or 21-236 Min. grade B or 21-356 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B)

21-765 Introduction to Parallel Computing and Scientific Computation

Spring: 9 units
Course objectives: to develop structural intuition of how the hardware and the software work, starting from simple systems to complex shared resource architectures; to provide guidelines about how to write and document a software package; to familiarize the audience with the main parallel programming techniques and the common software packages/libraries. (One 110 minute lecture)

Course Website: http://www.math.cmu.edu/~florin/M21-765/index.html (http://www.math.cmu.edu/~florin/M21-765/)

21-770 Introduction to Continuum Mechanics

Intermittent: 12 units
General discussion of the behavior of continuous bodies with an emphasis on those concepts common to the description of all continuous bodies. Specific examples from elasticity and fluid mechanics. Familiarity with analysis in finite dimensional spaces will be assumed. (Three 50 minute lectures)
Prerequisites: (21-456 Min. grade B or 21-236 Min. grade B or 21-356 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-800 Advanced Topics in Logic

Intermittent: 12 units
This course affords students with the opportunity to study topics which are in the area of expertise of the instructor. This course may taken more than once if content is sufficiently different. Course prerequisites will depend on the content of the course. Please see the course URL for semester-specific topics. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-455 Min. grade B or 21-235 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

Course Website: https://www.cmu.edu/math/courses/special-topics.html

21-801 Advanced Topics in Discrete Mathematics

Intermittent: 12 units
Course topics will vary depending on the semester and instructor. May be taken more than once if content is sufficiently different. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-235 Min. grade B or 21-455 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

21-803 Model Theory III

Spring: 12 units
We will concentrate in classification theory for first-order theories. The theory was developed mostly by Saharon Shelah presented in his 1978 (2nd ed 1990) book and in several hundreds of papers. We will present a modern overview of Shelah's theory incorporating few recent innovations and simplifications. The development of the theory was motivated by set-theoretic questions like: "what is the asymptotic behavior of the function I(/aleph_/alpha,T) as a function of /alpha ?" and "what is the first /lambda such that an uncountable first-order stable theory T is stable in /lambda?" Surprisingly the full answer to such combinatorial set-theoretic questions led for a development and discovery of a conceptually rich theory which seems to be related to aspects of commutative algebra and algebraic-geometry. This theory found several applications in the form of solving fundamental problems in classical fields of mathematics among them geometry and number theory. The focus will be on the simplest and most fundamental aspects of the pure theory. Primarily around a notion called forking and various characterizations of classes of theories. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-235 Min. grade B or 21-455 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B) and 21-703 Min. grade B

21-820 Advanced Topics in Analysis

Intermittent: 12 units
Course topics will vary depending on the semester and instructor. May be taken more than once if content is sufficiently different.
Prerequisites: (21-456 Min. grade B or 21-236 Min. grade B or 21-356 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B)

Course Website: https://www.cmu.edu/math/courses/special-topics.html

21-830 Advanced Topics in Partial Differential Equations

Intermittent: 12 units
Course topics will vary depending on the semester and instructor. May be taken more than once if content is sufficiently different.
Prerequisites: (21-355 Min. grade B or 21-455 Min. grade B or 21-235 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B) and 21-632 Min. grade B

21-832 Partial Differential Equations II

Intermittent: 12 units
Elliptic boundary value problems, Green's theorem calculations, integral equation methods, variational formulations and Galerkin's method, regularity theory, parabolic problems and semigroups. (Three 50 minute lectures)
Prerequisites: (21-236 Min. grade B or 21-356 Min. grade B or 21-456 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B) and 21-732 Min. grade B

21-849 Special Topics

Intermittent: 12 units
This course affords students with the opportunity to study topics which are in the area of expertise of the instructor. This course may taken more than once if content is sufficiently different. Course prerequisites will depend on the content of the course. Please see the course URL for semester-specific topics. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-455 Min. grade B or 21-235 Min. grade B) and 21-373 Min. grade B and 21-721 Min. grade B

Course Website: https://www.cmu.edu/math/courses/special-topics.html

21-860 Advanced Topics In Numerical Analysis

Intermittent: 12 units
Content varies. May be taken more than once if content is sufficiently different. (Three 50 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-455 Min. grade B or 21-235 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B) and 21-660 Min. grade B

21-880 Stochastic Calculus

Fall: 12 units
This is a first Ph.D.-level course in stochastic calculus for continuous-time processes. It includes martingales and semi-martingales, Brownian motion, the Poisson process, representation of continuous martingales as time-changed Brownian motions, construction of the Ito integral, and Ito's formula. (Two 80 minute lectures)
Prerequisites: (21-355 Min. grade B or 21-455 Min. grade B or 21-235 Min. grade B) and (21-237 Min. grade B or 21-373 Min. grade B) and 21-721 Min. grade B

21-882 Advanced Topics in Financial Mathematics

Intermittent: 12 units
Content varies. May be taken more than once if content is sufficiently different. (Two 80 minute lectures)
Prerequisites: (21-235 Min. grade B or 21-355 Min. grade B or 21-455 Min. grade B) and (21-373 Min. grade B or 21-237 Min. grade B) and 21-721 Min. grade B

21-901 Master's Degree Research

All Semesters
This course is for students admitted to the Mathematical Sciences Honors Degree Program. It allows for students to engage in research activities related to their Master's thesis, under the supervision of their thesis supervisor. The supervisor should be contacted prior to enrollment in 21-901, as they are required to provide consent for the student's enrollment to the Academic Program Coordinator, who authorizes enrollment in this course.