Department of Mathematical Sciences Courses

Note on Course Numbers

Each Carnegie Mellon course number begins with a two-digit prefix which designates the department offering the course (76-xxx courses are offered by the Department of English, etc.). Although each department maintains its own course numbering practices, typically the first digit after the prefix indicates the class level: xx-1xx courses are freshmen-level, xx-2xx courses are sophomore level, etc. xx-6xx courses may be either undergraduate senior-level or graduate-level, depending on the department. xx-7xx courses and higher are graduate-level. Please consult the Schedule of Classes (https://enr-apps.as.cmu.edu/open/SOC/SOCServlet) each semester for course offerings and for any necessary pre-requisites or co-requisites.

21-101 Freshman Mathematics Seminar
Fall: 3 units
This course is offered in the Fall semester for first semester Freshmen interested in majoring in mathematics. Topics vary from year to year. Recent topics have included Fermat's last theorem, finite difference equations, convexity, and fractals. 3 hrs. lec.

21-105 Pre-Calculus
Summer: 9 units
Review of basic concepts, logarithms, functions and graphs, inequalities, polynomial functions, complex numbers, and trigonometric functions and identities. Special summer program only. 3 hrs lec., 1 hr.rec.

21-111 Differential Calculus
Fall and Spring: 10 units
Review of basic algebra, functions, limits, derivatives of algebraic, exponential and logarithmic functions, curve sketching, maximum-minimum problems. Successful completion of 21-111 and 21-112 entitles a student to enroll in any mathematics course for which 21-120 is a prerequisite. 3 hrs. lec., 2 hrs. rec.

21-112 Integral Calculus
Fall and Spring: 10 units
Definite and indefinite integrals, and hyperbolic functions; applications of integration, integration by substitution and by parts. Successful completion of 21-111 and 21-112 entitles a student to enroll in any mathematics course for which 21-120 is a prerequisite. 3 hrs. lec., 2 hrs. rec.
Prerequisite: 21-111

21-115 Basic Differential Calculus
Summer: 5 units
Functions, limits, derivatives, curve sketching, Mean Value Theorem, trigonometric functions, related rates, linear and quadratic approximations, maximum-minimum problems. Special summer program only.

21-120 Differential and Integral Calculus
All Semesters: 10 units
Functions, limits, derivatives, logarithmic, exponential, and trigonometric functions, inverse functions; L'Hospital's Rule, curve sketching, Mean Value Theorem, related rates, linear and quadratic approximations, maximum-minimum problems, inverse functions, definite and indefinite integrals, and hyperbolic functions; applications of integration, integration by substitution and by parts. 3 hrs lec., 2 hrs. rec.

21-122 Integration and Approximation
Fall and Spring: 10 units
Integration by trigonometric substitution and partial fractions; arclength; improper integrals; Simpson's and Trapezoidal Rules for numerical integration; separable differential equations, Newton's method, Euler's method, Taylor's Theorem including a discussion of the remainder, sequences, series, power series. Parametric curves, polar coordinates, vectors, dot product. 3 hrs lec., 2 hrs. rec.
Prerequisites: 21-112 or 21-120

21-124 Calculus II for Biologists and Chemists
Spring: 10 units
This is intended as a second calculus course for biology and chemistry majors. It uses a variety of computational techniques based around the use of MATLAB or a similar system. Topics to be covered include: Integration: techniques and numerical integration. Ordinary differential equations: techniques for solving ODEs and numerical methods. Modeling with ODEs (e.g., infection, population models). Linear algebra: matrices, complex numbers, eigenvalues, eigenvectors. Systems of ordinary differential equations (if time allows: stability of differential systems). Probability: discrete and continuum probability, conditional probability and independence, limit theorems, important distributions, probabilistic models. 3 hrs. lec., 2 hrs. rec. Prerequisite: 21-112 or 21-120.
Prerequisites: 21-112 or 21-120

21-126 Introduction to Mathematical Software
Spring: 3 units
This course provides an introduction to the use of several software packages, which are useful to mathematics students. Among the packages are Maple and Mathematica for symbolic computing, TeX and LaTeX for mathematical documents, and Matlab for numerical computing. The course will also introduce the mathematical facilities built into spreadsheets such as Excel. The aim of the course is to provide the student with some basic skills in the use of this software without attempting complete coverage. A deeper knowledge of the software will be easy to obtain after completing this course. There are no prerequisites for the course, other than basic computer literacy and a knowledge of elementary mathematics. It is suggested that the course should be taken during the first two years of undergraduate studies.

21-127 Concepts of Mathematics
Fall and Spring: 10 units
This course introduces the basic concepts, ideas and tools involved in doing mathematics. As such, its main focus is on presenting informal logic, and the methods of mathematical proof. These subjects are closely related to the application of mathematics in many areas, particularly computer science. Topics discussed include a basic introduction to elementary number theory, induction, the algebra of sets, relations, equivalence relations, congruences, partitions, and functions, including injections, surjections, and bijections. A basic introduction to the real numbers, rational and irrational numbers. Supremum and infimum of a set. 3 hrs. lec., 2 hrs. rec.

21-128 Mathematical Concepts and Proofs
Intermittent: 12 units
This course is intended for MCS first-semester students who are interested in pursuing a major in mathematical sciences. The course introduces the basic concepts, ideas and tools involved in doing mathematics. As such, its main focus in on presenting informal logic, and the methods of mathematical proof. These subjects are closely related to the application of mathematics in many areas, particularly computer science. Topics discussed include a basic introduction to elementary number theory, induction, the algebra of sets, relations, equivalence relations, congruences, partitions, and functions, including injections, surjections, and bijections. A basic introduction to the real numbers, rational and irrational numbers. Supremum and infimum of a set. This course is a superset of 21-127, with additional out of class time devoted to proofs and additional topics in math. 3 hrs. lec., 2 hrs. rec.

21-228 Discrete Mathematics
Spring: 9 units
The techniques of discrete mathematics arise in every application of mathematics, which is not purely continuous, for example in computer science, economics, and general problems of optimization. This course introduces two of the fundamental areas of discrete mathematics: enumeration and graph theory. The introduction to enumeration includes permutations, combinations, and topics such as discrete probability, combinatorial distributions, recurrence relations, generating functions, Ramsey's Theorem, and the principle of inclusion and exclusion. The introduction to graph theory includes topics such as paths, walks, connectivity, Eulerian and Hamilton cycles, planar graphs, Euler's Theorem, graph coloring, matchings, networks, and trees. 3 hrs. lec, 1 hr. rec.
Prerequisites: 21-128 or 21-127

21-235 Mathematical Studies Analysis I
Fall: 12 units
An honors version of 21-355 for students of greater aptitude and motivation. Topics to be covered include: The Real Number System: sups and infs, completeness, integers and rational numbers. Metric spaces, normed spaces, inner product spaces and their specialization to the Euclidean space. Topological properties of metric spaces (open sets, closed sets, density, compactness, Heine-Borel Theorem). Sequences and convergence; completeness. Baire Category Theorem. Real sequences: limsup and liminf, subsequences, monotonic sequences, Bolzano-Weierstrass Theorem. Real series (criteria for convergence). Continuity, limits of functions, attainment of extrema, Intermediate Value Theorem, uniform continuity. Differentiation of functions of one variable: Chain Rule, local extrema, Mean-Value Theorems, L'Ho'pital's Rule, Taylor's Theorem. Riemann Integration: Partitions, upper and lower integrals, sufficient conditions for integrability, Fundamental Theorem of Calculus. 3 hrs. lec.
Prerequisites: (21-127 or 21-128) and 21-269

21-236 Mathematical Studies Analysis II
Spring: 12 units
An honors version of 21-356 for students of greater aptitude and motivation. Topics to be covered include: Vector differential calculus: differentiability, partial derivatives, directional derivatives, gradients, Jacobians, the chain rule, implicit function theorem, inverse function theorem. Local extrema, constrained problems (Lagrange multipliers). Integration of differential forms: Manifolds, Differential forms (properties, differentiation, change of variables), partition of unity, integration, volume form, area form, Stokes' theorem. Sequences of Functions: Pointwise convergence, uniform convergence, Arzela-Ascoli, Weierstrass approximation theorem. Series of functions: Power series, Fourier series, orthonormal bases. 3 hrs. lec.
Prerequisites: 21-242 and 21-235

21-237 Mathematical Studies Algebra I
Fall: 12 units
An honors version of 21-373 Algebraic structures for students of greater aptitude and motivation. Abstract algebra is the study of algebraic systems by the axiomatic method, and it is one of the core areas of modern mathematics. This course is a rigorous and fast-paced introduction to the basic objects in abstract algebra. Topics to be covered include: Homomorphisms. Subgroups, cosets, Lagrange's theorem. Conjugation. Normal subgroups, quotient groups, first isomorphism theorem. Automorphisms, the automorphism group, characteristic subgroups. Group actions, Cauchy's Theorem, Sylow's theorem. Normalisers and centralisers, class equation, finite p-groups. Dihedral and alternating groups. The second and third isomorphism theorems. Simple groups, statement of Jordan-Holder theorem, semidirect product of groups. Subrings, ideals, quotient rings, first isomorphism theorem. Polynomial rings. Zorn's Lemma. Prime and maximal ideals, prime and irreducible elements. PIDs and UFDs. Noetherian domains. Hilbert Basis Theorem. Gauss' lemma. Eisenstein criterion. Field of fractions of an integral domain. k a field implies k[x] a PID, R a UFD implies R[x] a UFD. Finite fields and applications. 3 hrs. lec.
Prerequisites: (21-127 or 21-128) and 21-269

21-238 Mathematical Studies Algebra II
Spring: 12 units
An honors version of 21-341 Linear Algebra for students of greater aptitude and motivation. Linear algebra is a crucial tool in pure and applied mathematics. This course aims to introduce the main ideas at a high level of rigour and generality. The course starts with the study of (potentially) infinite-dimensional vector spaces over an arbitrary field, continues with the theory of modules (where the role of the field is now played by an arbitrary ring), and concludes with the development of real and complex inner product spaces. Topics to be covered include: Review of fields. Review of Zorn's Lemma. Vector spaces (possibly in finite dimensional) over an arbitrary field. Independent sets, bases, existence of a basis, exchange lemma, dimension. Linear transformations, dual space. Multilinear maps, tensor product, exterior power, determinant of a transformation. Eigenvalues, eigenvectors, characteristic and minimal polynomial of a transformation, Cayley-Hamilton theorem. Review of commutative rings. R-modules. Sums and quotients of modules. Free modules. Structure theorem for fg modules over a PID and applications (Jordan and rational canonical form, structure theory of fg abelian groups). Review of real and complex numbers. Real and complex inner product spaces. Orthonormal sets, orthonormal bases, Gram-Schmidt. Examples: F^n and l^2(F) for F = R; C. Operators: Symmetric/Hermitian and Orthogonal/Unitary operators. Spectral theorem. Quadratic forms. Singular value decomposition. Possible additional topics (time permitting): applications to combinatorics, category theory, representations of finite groups, normed spaces. 3 hrs. lec.
Prerequisites: 21-242 and 21-237

21-240 Matrix Algebra with Applications
Fall and Spring: 10 units
Vectors and matrices, the solution of linear systems of equations, vector spaces and subspaces, orthogonality, determinants, real and complex eigenvalues and eigenvectors, linear transformations. The course is intended for students in Economics, Statistics, Information Systems, and it will focus on topics relevant to these fields. 3 hrs. lec., 1 hr. rec.

21-241 Matrices and Linear Transformations
Fall and Spring: 10 units
A first course in linear algebra intended for scientists, engineers, mathematicians and computer scientists. Students will be required to write some straightforward proofs. Topics to be covered: complex numbers, real and complex vectors and matrices, rowspace and columnspace of a matrix, rank and nullity, solving linear systems by row reduction of a matrix, inverse matrices and determinants, change of basis, linear transformations, inner product of vectors, orthonormal bases and the Gram-Schmidt process, eigenvectors and eigenvalues, diagonalization of a matrix, symmetric and orthogonal matrices. 21-127 is strongly recommended. 3 hrs. lec., 1 hr. rec

21-242 Matrix Theory
Fall and Spring: 10 units
An honors version of 21-241 (Matrix Algebra and Linear Transformations) for students of greater aptitude and motivation. More emphasis will be placed on writing proofs. Topics to be covered: complex numbers, real and complex vectors and matrices, rowspace and columnspace of a matrix, rank and nullity, solving linear systems by row reduction of a matrix, inverse matrices and determinants, change of basis, linear transformations, inner product of vectors, orthonormal bases and the Gram-Schmidt process, eigenvectors and eigenvalues, diagonalization of a matrix, symmetric and orthogonal matrices, hermitian and unitary matrices, quadratic forms. 3 hrs. lec., 1 hr. rec.

21-256 Multivariate Analysis
Fall and Spring: 9 units
This course is designed for students in Economics or Business Administration. Matrix algebra: vectors, matrices, systems of equations, dot product, cross product, lines and planes. Optimization: partial derivatives, the chain rule, gradient, unconstrained optimization, constrained optimization (Lagrange multipliers and the Kuhn-Tucker Theorem). Improper integrals. Multiple integration: iterated integrals, probability applications, triple integrals, change of variables. 3 hrs lec., 1 hr rec.
Prerequisites: 21-112 or 21-120

21-257 Models and Methods for Optimization
Fall and Spring: 9 units
Introduces basic methods of operations research and is intended primarily for Business Administration and Economics majors. Review of linear systems; linear programming, including the simplex algorithm, duality, and sensitivity analysis; the transportation problem; the critical path method; the knapsack problem, traveling salesman problem, and an introduction to set covering models. 3 hrs. lec., 1 hr. rec.
Prerequisites: 21-242 or 21-256 or 21-241 or 21-240 or 18-202 or 06-262

21-259 Calculus in Three Dimensions
Fall and Spring: 9 units
Vectors, lines, planes, quadratic surfaces, polar, cylindrical and spherical coordinates, partial derivatives, directional derivatives, gradient, divergence, curl, chain rule, maximum-minimum problems, multiple integrals, parametric surfaces and curves, line integrals, surface integrals, Green-Gauss theorems. 3 hrs. lec., 1 hr. rec.
Prerequisite: 21-122

21-260 Differential Equations
Fall and Spring: 9 units
Ordinary differential equations: first and second order equations, applications, Laplace transforms; partial differential equations: partial derivatives, separation of variables, Fourier series; systems of ordinary differential equations; applications. 21-259 or 21-268 or 21-269 are recommended. 3 hrs. lec., 1 hr. rec.
Prerequisite: 21-122

21-261 Introduction to Ordinary Differential Equations
Spring: 10 units
A first course in ordinary differential equations intended primarily for math majors and for those students interested in a more conceptual treatment of the subject. One of the goals of this course is to prepare students for upper level courses on differential equations, mathematical analysis and applied mathematics. Students will be required to write rigorous arguments. Topics to be covered: Ordinary differential equations: first and second order equations, applications, Laplace transform, systems of linear ordinary differential equations; systems of nonlinear ordinary differential equations, equilibria and stability, applications. Note: courses 21-259, or 21-268, or 21-269 are recommended. 21-128 can replace 21-127 as a corequisite. 3 hrs. lec., 1 hr. rec.
Prerequisite: 21-122

21-268 Multidimensional Calculus
Fall and Spring: 10 units
A serious introduction to multidimensional calculus that makes use of matrices and linear transformation. Results will be stated carefully and rigorously. Students will be expected to write some proofs; however, some of the deeper results will be presented without proofs. Topics to be covered include: functions of several variables, regions and domains, limits and continuity, partial derivatives, linearization and Jacobian matrices, chain rules, inverse and implicit functions, geometric applications, higher derivatives, Taylor's theorem, optimization, vector fields, multiple integrals and change of variables, Leibnitz's rule, line integrals, Green's theorem, path independence and connectedness, conservative vector fields, surfaces and orientability, surface integrals, divergence theorem and Stokes's theorem. 3 hrs. lec.
Prerequisites: 21-122 and (21-241 or 21-242)

21-269 Vector Analysis
Spring: 10 units
An honors version of 21-268 for students of greater aptitude and motivation. More emphasis will be placed on writing proofs. Topics to be covered include: basic geometry and topology of Euclidean space, curves in space, arclength, curvature and torsion, functions on Euclidean spaces, limits and continuity, partial derivatives, gradients and linearization, chain rules, inverse and implicit function theorems, geometric applications, higher derivatives, Taylor's theorem, optimization, vector fields, multiple integrals and change of variables, Leibnitz's rule, conservative and solenoidal vector fields, divergence and curl, surfaces and orientability, surface integrals, Gauss-Green theorems and Stokes's theorem. A grade of B or better in 21-242 is required. 3 hrs. lec.
Prerequisites: 21-122 and 21-242 Min. grade B

21-270 Introduction to Mathematical Finance
Spring: 9 units
This is a first course for those considering majoring or minoring in Computational Finance. The theme of this course is pricing derivative securities by replication. The simplest case of this idea, static hedging, is used to discuss net present value of a non-random cash flow, internal rate of return, and put-call option parity. Pricing by replication is then considered in a one-period random model. Risk-neutral probability measures, the Fundamental Theorems of Asset Pricing, and an introduction to expected utility maximization and mean-variance analysis are presented in this model. Finally, replication is studied in a multi-period binomial model. Within this model, the replicating strategies for European and American options are determined. 3 hrs. lec.
Prerequisites: 21-120 or 21-112

21-272 Introduction to Partial Differential Equations
Fall: 9 units
A Partial Differential Equation (PDE for short) is a differential equation involving derivatives with respect to more than one variable. These arise in numerous applications from various disciplines. A prototypical example is the heat equation, governing the evolution of temperature in a conductor. This course will serve as a first introduction to PDE's, and will focus on the most important model equations. It will cover both analytical methods (e.g. separation of variables, Green's functions), numerical methods (e.g. finite elements) and the use of a computer to approximate and visualize solutions. Time permitting, it will touch upon the mathematical ideas behind phenomena observed in nature (e.g. speed of wave propagation, and/or shocks in traffic flow).
Prerequisites: (21-269 or 21-268 or 21-259) and (21-260 or 21-261 or 33-231)

21-292 Operations Research I
Spring: 9 units
Operations research offers a scientific approach to decision making, most commonly involving the allocation of scarce resources. This course develops some of the fundamental methods used. Linear programming: the simplex method and its linear algebra foundations, duality, post-optimality and sensitivity analysis; the transportation problem; the critical path method; non-linear programming methods. 3 hrs. lec., 1 hr. rec.
Prerequisites: 21-122 and (21-241 or 21-242)

21-295 Putnam Seminar
Fall: 3 units
A problem solving seminar designed to prepare students to participate in the annual William Lowell Putnam Mathematical Competition. Students solve and present their solutions to problems posed.

21-296 Millennium Problems Seminar
Intermittent: 3 units
This seminar course will discuss some of the most important unsolved problems of mathematics (as deemed in 2000 by an international committee of mathematicians): The Riemann Hypothesis; Yang-Mills Theory and the Mass Gap Hypothesis; the P. vs. NP Problem; smoothness of solutions of the Navier-Stokes Equations; the Hodge Conjecture; the Birch and Swinnerton-Dyer Conjecture. If the time allows, the Poincare conjecture will also be discussed. 1 hr. lec.

21-300 Basic Logic
Fall: 9 units
Propositional and predicate logic: Syntax, proof theory and semantics up to completeness theorem, Lowenheim Skolem theorems, and applications of the compactness theorem. 3 hrs. lec.
Prerequisites: 21-373 or 15-251 or 21-228

21-301 Combinatorics
Fall and Spring: 9 units
A major part of the course concentrates on algebraic methods, which are relevant in the study of error correcting codes, and other areas. Topics covered in depth include permutations and combinations, generating functions, recurrence relations, the principle of inclusion and exclusion, and the Fibonacci sequence and the harmonic series. Additional topics may include existence proofs, partitions, finite calculus, generating combinatorial objects, Polya theory, codes, probabilistic methods. 3 hrs. lec
Prerequisites: 21-122 and (15-251 or 21-228)

21-302 Lambda Calculus
Spring: 9 units
An introductory course in classical lambda calculus, with an emphasis on syntax. The course will describe many research problems which are suitable topics for senior theses or master's theses. Topics will include: Basic properties of reduction and conversion; Reduction and conversion strategies; Calculability and representation of data types; Elementary theory of Ershov numberings; Bohm's theorem, easy terms, and other exotic combinations; Solvability of functional equations (unification); Combinatorics and bases; Simple and algebraic types; Labelled reduction and intersection types; Extensionality and the omega rule.
Prerequisites: (15-150 or 80-310 or 21-300) and 21-301

21-320 Symbolic Programming Methods
Spring: 9 units
The objective of this course is to learn to program in Maple, a powerful symbolic mathematics package available on many platforms at Carnegie Mellon. After learning what Maple can do with the commands provided with the package, students will learn to develop their own Maple functions to accomplish extended mathematical computations. Grades in the course will be based mostly on project work. Projects may come from any relevant field and may be graphical, numerical, or symbolic or all three. The course will involve online demonstrations in most classes. 3 hrs. lec.
Prerequisites: (21-127 or 21-128) and 21-122

21-325 Probability
Fall: 9 units
This course focuses on the understanding of basic concepts in probability theory and illustrates how these concepts can be applied to develop and analyze a variety of models arising in computational biology, finance, engineering and computer science. The firm grounding in the fundamentals is aimed at providing students the flexibility to build and analyze models from diverse applications as well as preparing the interested student for advanced work in these areas. The course will cover core concepts such as probability spaces, random variables, random vectors, multivariate densities, distributions, expectations, sampling and simulation; independence, conditioning, conditional distributions and expectations; limit theorems such as the strong law of large numbers and the central limit theorem; as well as additional topics such as large deviations, random walks and Markov chains, as time permits. 3 hrs. lec.
Prerequisites: 21-259 or 21-268 or 21-269

21-329 Set Theory
Spring: 9 units
Set theory was invented about 110 years ago by George Cantor as an instrument to understand infinite objects and to compare different sizes of infinite sets. Since then set theory has come to play an important role in several branches of modern mathematics, and serves as a foundation of mathematics. Contents: Basic properties of natural numbers, countable and uncountable sets, construction of the real numbers, some basic facts about the topology of the real line, cardinal numbers and cardinal arithmetic, the continuum hypothesis, well ordered sets, ordinal numbers and transfinite induction, the axiom of choice, Zorn's lemma. Optional topics if time permits: Infinitary combinatorics, filters and large cardinals, Borel and analytic sets of reals. 3 hrs. lec.
Prerequisites: 21-128 or 21-127

21-341 Linear Algebra
Fall and Spring: 9 units
21-341 Linear Algebra. A mathematically rigorous treatment of Linear Algebra over an arbitrary field. Topics studied will include abstract vector spaces, linear transformations, determinants, eigenvalues, eigenvectors, inner products, invariant subspaces, canonical forms, the spectral theorem and the singular value decomposition. 21-373 recommended. 3 hrs. lec.
Prerequisites: (21-241 and 21-373) or 21-242

21-350 History of Mathematics
Intermittent: 9 units
Mathematics has a long and interesting history, and there is much insight into both mathematics and history to be gained from its study. The emphasis here will be on learning the mathematics with the added value of appreciating it in historical context. Selected topics may range from early number systems, the development of geometry, the emergence of the ideas of analysis, through to the origins of modern set theory. 3 hrs. lec.

21-355 Principles of Real Analysis I
Fall and Spring: 9 units
This course provides a rigorous and proof-based treatment of functions of one real variable. The Real Number System: Field and order axioms, sups and infs, completeness, integers and rational numbers. Real Sequences: Limits, cluster points, limsup and liminf, subsequences, monotonic sequences, Cauchy's criterion, Bolzano-Weierstrass Theorem. Topology of the Real Line: Open sets, closed sets, density, compactness, Heine-Borel Theorem. Continuity: attainment of extrema, Intermediate Value Theorem, uniform continuity. Differentiation: Chain Rule, local extrema, Mean-Value Theorems, L'Hospital's Rule, Taylor's Theorem. Riemann Integration: Partitions, upper and lower integrals, sufficient conditions for integrability, Fundamental Theorem of Calculus. Sequences of Functions: Pointwise convergence, uniform convergence, interchanging the order of limits. The course presumes some mathematical sophistication including the ability to recognize, read, and write proofs. 3 hrs lec.
Prerequisites: (21-128 or 21-127) and 21-122

21-356 Principles of Real Analysis II
Spring: 9 units
This course provides a rigorous and proof-based treatment of functions of several real variables. Topology in metric spaces, specialization to finite dimensional normed linear spaces. Vector differential calculus: continuity and the total derivative, partial derivatives, directional derivatives, gradients, Jacobians, the chain rule, implicit function theorem. Vector integral calculus: double and triple integrals, arclength and surface area, line integrals, Green's Theorem, surface integrals, Divergence and Stokes Theorems. If time permits: trigonometric series, Fourier series for orthonormal bases, minimization of square error. The course presumes some mathematical sophistication including the ability to recognize, read, and write proofs. 21-268 or 21-269 are strongly recommended rather than 21-259. 3 hrs lec.
Prerequisites: (21-259 or 21-269 or 21-268) and 21-241 and 21-355

21-360 Differential Geometry of Curves and Surfaces
Intermittent: 9 units
The course is a rigorous introduction to the differential and integral calculus of curves and surfaces. Topics to be covered include: Parameterized and regular curves Frenet equations canonical coordinate system, local canonical forms, global properties of plane curves Regular surfaces, differential functions on surfaces, the tangent plane and differential of a map, orientation of surfaces, characterization of compact orientable surfaces, classification of compact surfaces The geometry of the Gauss map, isometries and conformal maps, parallel transport, geodesics, the Gauss-Bonnet theorem and applications. More topics may be covered, as time allows. Students should be prepared to write proofs and perform computations. 21-356 or 21-236 are recommended. 3 hrs. lec.
Prerequisites: 21-269 or 21-268

21-365 Projects in Applied Mathematics
Intermittent: 9 units
This course provides students with an opportunity to solve problems posed by area companies. It is also designed to provide experience working as part of a team to solve problems for a client. The background needed might include linear programming, simulation, data analysis, scheduling, numerical techniques, etc.

21-366 Topics in Applied Mathematics
Intermittent: 9 units
Typical of courses that might be offered from time to time are game theory, non-linear optimization, and dynamic programming. Prerequisites will depend on the content of the course. 3 hrs. lec.
Prerequisites: 21-128 or 21-127

21-369 Numerical Methods
Fall and Spring: 9 units
This course provides an introduction to the use of computers to solve scientific problems. Methods for the computational solution of linear algebra systems, nonlinear equations, the interpolation and approximation of functions, differentiation and integration, and ordinary differential equations. Analysis of roundoff and discretization errors and programming techniques. 21-268 or 21-269 are recommended rather than 21-259. 3 hrs. lec.
Prerequisites: (15-110 or 15-112) and (21-259 or 21-269 or 21-268) and (21-240 or 21-241 or 21-242) and (21-260 or 33-231 or 21-261)

21-370 Discrete Time Finance
Fall: 9 units
This course introduces the Black-Scholes option pricing formula, shows how the binomial model provides a discretization of this formula, and uses this connection to fit the binomial model to data. It then sets the stage for Continuous-Time Finance by discussing in the binomial model the mathematical technology of filtrations, martingales, Markov processes and risk-neutral measures. Additional topics are American options, expected utility maximization, the Fundamental Theorems of Asset Pricing in a multi-period setting, and term structure modeling, including the Heath-Jarrow-Morton model. Students in 21-370 are expected to read and write proofs. 3 hrs lec.
Prerequisites: (21-270 or 70-492) and (21-259 or 21-268 or 21-269 or 21-256)

21-371 Functions of a Complex Variable
Intermittent: 9 units
This course provides an introduction to one of the basic topics of both pure and applied mathematics and is suitable for those with both practical and theoretical interests. Algebra and geometry of complex numbers; complex differentiation and integration. Cauchy's theorem and applications; conformal mapping; applications. 21-268 or 21-269 are recommended rather than 21-259. 3 hrs. lec.
Prerequisites: 21-355 or 21-235

21-372 Partial Differential Equations and Fourier Analysis
Intermittent: 9 units
This course provides an introduction to partial differential equations and is recommended for majors in mathematics, physical science, or engineering. Boundary value problems on an interval, Fourier series, uniform convergence, the heat, wave, and potential equations on bounded domains, general theory of eigenfunction expansion, the Fourier integral applied to problems on unbounded domains, introduction to numerical methods. 21-268 and 21-269 are recommended rather than 21-259; and 21-261 is recommended rather than 21-260. 3 hrs. lec.
Prerequisites: (21-268 or 21-269 or 21-259) and (21-260 or 21-261)

21-373 Algebraic Structures
Fall and Spring: 9 units
Groups: Homomorphisms. Subgroups, cosets, Lagrange's theorem. Conjugation. Normal subgroups, quotient groups, first isomorphism theorem. Group actions, Cauchy's Theorem. Dihedral and alternating groups. The second and third isomorphism theorems. Rings: Subrings, ideals, quotient rings, first isomorphism theorem. Polynomial rings. Prime and maximal ideals, prime and irreducible elements. PIDs and UFDs. Noetherian domains. Gauss' lemma. Eisenstein criterion. Fields: Field of fractions of an integral domain. Finite fields. Applications to coding theory, cryptography, number theory. 3 hrs lec.
Prerequisites: (21-128 or 21-127) and (21-241 or 21-242)

21-374 Field Theory
Spring: 9 units
The purpose of this course is to provide a successor to Algebraic Structures, with an emphasis on applications of groups and rings within algebra to some major classical problems. These include constructions with a ruler and compass, and the solvability or unsolvability of equations by radicals. It also offers an opportunity to see group theory and basic ring theory "in action", and introduces several powerful number theoretic techniques. The basic ideas and methods required to study finite fields will also be introduced. These ideas have recently been applied in a number of areas of theoretical computer science including primality testing and cryptography. 3 hrs. lec.
Prerequisite: 21-373

21-377 Monte Carlo Simulation for Finance
Intermittent: 9 units
first course in Monte Carlo simulation, with applications to Mathematical Finance. Students will put into practice many of the theoretical ideas introduced in Continuous Time Finance. Topics to be covered: random variable/stochastic process generation; options pricing; variance reduction; Markov chain Monte Carlo Methods.
Prerequisites: 21-325 Min. grade B or 21-420

21-378 Mathematics of Fixed Income Markets
Fall: 9 units
A first course in fixed income. Students will be introduced to the most common securities traded in fixed income markets and the valuation methods used to price them. Topics covered include discount factors; interest rates basics; pricing of coupon bonds; identifying the yield to maturity, as well as bond sensitivities to interest rates; term structure modeling; forward and swap rates; fixed income derivatives (including mortgage backed securities) and their valuation through backwards induction; fixed income indexes and return attribution. For a co-requisite, 36-225 can be accepted as an alternative for 21-325.
Prerequisite: 21-270 Min. grade B

21-380 Introduction to Mathematical Modeling
Intermittent: 9 units
This course shall examine mathematical models, which may be used to describe natural phenomena. Examples, which have been studied include: continuum description of highway traffic, discrete velocity models of a monotonic gas, chemotactic behavior in biological systems, European options pricing, and cellular-automata. Systems such as the first four are described by partial differential equations; the last involves discrete-time and discrete-phase dynamical systems, which have been used to successfully represent both physical and biological systems. The course will develop these models and then examine the behavior of the underlying systems, both analytically and numerically. The mathematical tools required will be developed in the course.
Prerequisites: (21-241 or 21-242) and (21-261 or 21-260)

21-393 Operations Research II
Fall: 9 units
Building on an understanding of Linear Programming developed in 21-292 Operations Research I, this course introduces more advanced topics. Integer programming, including cutting planes and branch and bound. Dynamic programming. An introduction to Combinatorial Optimization including optimal spanning trees, shortest paths, the assignment problem and max-flow/min-cut. The traveling salesman problem and NP-completeness. An important goal of this course is for the student to gain experience with the process of working in a group to apply operations research methods to solve a problem. A portion of the course is devoted to a group project based upon case studies and the methods presented. 36-410 recommended. 3 hrs. lec.
Prerequisites: (15-251 or 21-228) and 21-292

21-400 Intermediate Logic
Spring: 9 units
The course builds on the proof theory and model theory of first-order logic covered in 21-300. These are applied in 21-400 to Peano Arithmetic and its standard model, the natural numbers. The main results are the incompleteness, undefinability and undecidability theorems of Godel, Tarski, Church and others. Leading up to these, it is explained how logic is formalized within arithmetic, how this leads to the phenomenon of self-reference, and what it means for the axioms of a theory to be computably enumerable. Related aspects of computability theory are included to the extent that time permits.
Prerequisite: 21-300

21-420 Continuous-Time Finance
Spring: 9 units
This course begins with Brownian motion, stochastic integration,and Ito's formula from stochastic calculus. This theory is used to develop the Black-Scholes option pricing formula and the Black-Scholes partial differential equation. Additional topics may include models of credit risk, simulation, and expected utility maximization. 3 hrs lec.
Prerequisites: (21-260 or 18-202) and 21-370 and (21-325 or 36-225 or 36-217)

21-435 Applied Harmonic Analysis
Spring: 9 units
This course serves as a broad introduction to harmonic analysis and its applications, particularly in 1-dimensional signal processing and in image processing, for undergraduate students in mathematics, engineering, and the applied sciences. Topics include: Discrete Fourier transform and fast Fourier transform; Fourier series and the Fourier transform; Hilbert spaces and applications; Shannon sampling theorem, bandlimited functions, uncertainty principle; Wavelets and multi-resolution analysis; Applications in image processing.
Prerequisites: (21-355 or 21-235) and (21-241 or 21-242)

21-441 Number Theory
Fall: 9 units
Number theory deals with the integers, the most basic structures of mathematics. It is one of the most ancient, beautiful, and well-studied branches of mathematics, and has recently found surprising new applications in communications and cryptography. Course contents: Structure of the integers, greatest common divisiors, prime factorization. Modular arithmetic, Fermat's Theorem, Chinese Remainder Theorem. Number theoretic functions, e.g. Euler's function, Mobius functions, and identities. Diophantine equations, Pell's Equation, continued fractions. Modular polynomial equations, quadratic reciprocity. 3 hrs. lec.
Prerequisites: (21-242 or 21-241) and 21-373

21-465 Topology
Fall: 9 units
Metric spaces. Topological spaces. Separation axioms. Open, closed and compact sets. Continuous functions. Product spaces, subspaces, quotient spaces. Connectedness and path-connectedness. Homotopy. Fundamental group of a pointed space. Simply connected spaces. Winding number, the fundamental group of the circle. Functorial property of the fundamental group. Brouwer fixed point theorem. Covering spaces. van Kampen's theorem. 2-manifolds. Triangulations. Euler characteristic. Surgery, classification of compact 2-manifolds. 3 hrs lec.
Prerequisites: 21-355 and 21-373

21-467 Differential Geometry
9 units
This course will provide a thorough and rigorous introduction to differential geometry on manifolds. Contents: Differentiable manifolds; tangent spaces; vector fields and n-forms; integral curves; cotangent vectors; tensors; Riemannian metrics; connection; parallel transport; geodesics and convex neighborhoods; sectional, Ricci, scalar curvatures; tensors on Riemannian manifolds; Lie groups; transformation groups.
Prerequisites: 21-373 and 21-356

21-469 Numerical Methods II: Scientific Computing
Spring: 9 units
This course is the continuation for 21-369 and is centered on the mathematics of scientific computing and advanced numerical methods. The focus of this course is on numerical methods for partial differential equations, with an emphasis on computing and applications. This course is intended for undergraduate students in mathematics, engineering, and the applied sciences. Topics will include: Numerical methods for dynamical systems; numerical methods for linear partial differential equations; computational linear algebra; data fitting and approximation; computational methods for nonlinear problems.
Prerequisites: (21-235 or 21-355) and (21-261 or 21-260) and (21-242 or 21-241) and 21-272 and 21-369

21-470 Selected Topics in Analysis
Intermittent: 9 units
Typical of courses, which are offered from time to time are finite difference equations, calculus of variations, and applied control theory. The prerequisites will depend on the content of the course. 3 hrs. lec.
Prerequisites: 21-241 and 21-259 and 21-260

21-476 Introduction to Dynamical Systems
Intermittent: 9 units
This course is an introduction to differentiable dynamical systems. The material includes basic properties of dynamical systems, including the existence and uniqueness theory, continuation, singular points, orbits, and their classification. The Poincare'-Bendixson theorem and typical applications, like Lienard equations and Lotka-Volterra are also covered. An introduction to chaos as time permits. 3 hrs. lec.
Prerequisites: (21-241 or 21-242) and 21-261

21-484 Graph Theory
Spring: 9 units
Graph theory uses basic concepts to approach a diversity of problems and nontrivial applications in operations research, computer science and other disciplines. It is one of the very few mathematical areas where one is always close to interesting unsolved problems. Topics include graphs and subgraphs, trees, connectivity, Euler tours and Hamilton cycles, matchings, graph colorings, planar graphs and Euler's Formula, directed graphs, network flows, counting arguments, and graph algorithms. 3 hrs. lec.
Prerequisites: (15-251 or 21-228) and (21-242 or 21-241)

21-499 Undergraduate Research Topic
Fall: 9 units
This course affords undergraduates to pursue elementary research topics in the area of expertise of the instructor. The prerequisites will depend on the content of the course.

21-590 Practicum
All Semesters
Students in this course gain experience with the application of mathematical models to business and/or industrial problems during an internship. The internship is set up by the student in consultation with a faculty member. The students must also have a mentor at the firm providing the internship, who together with the faculty member develops a description of the goals of the internship. The internship must include the opportunity to learn about problems which have mathematical content.

21-599 Undergraduate Reading and Research
Fall and Spring
Individual reading courses or projects in mathematics and its applications. Prerequisites and units to be negotiated with individual instructors.

21-600 Mathematical Logic I
Fall: 12 units
The study of formal logical systems, which model the reasoning of mathematics, scientific disciplines, and everyday discourse. Propositional Calculus and First-order Logic. Syntax, axiomatic treatment, derived rules of inference, proof techniques, computer-assisted formal proofs, normal forms, consistency, independence, semantics, soundness, completeness, Lowenheim-Skolem Theorem, compactness, equality. 3 hrs. lec.
Prerequisites: 21-373 Min. grade B or 21-484 Min. grade B or 21-228 Min. grade B

21-602 Introduction to Set Theory I
Fall: 12 units
First order definability and the Zermelo-Fraenkel axioms; cardinal arithmetic, ordered sets, well-ordered sets (axiom of choice), transfinite induction, the filter of closed unbounded sets (Fodor, Ulm and Solovay's theorems), Delta systems, basic results in partition calculus (e.g., Ramsey's Theorem and the Erdos-Rado Theorem); small to medium large cardinals; applications to general topology (e.g., Alexandroff's conjecture), and the basic ideas of descriptive set theory. The independence of Suslin conjecture from the usual axioms. Godel's axiom of constructibility. Time permitting, the Galvin-Hajnal-Shelah inequality will be proved. 3 hrs. lec.

21-603 Model Theory I
Intermittent: 12 units
Similarity types, structures; downward Lowenheim Skolem theorem; construction of models from constants, Henkin's omitting types theory, prime models; elementary chains of models, basic two cardinal theorems, saturated models, basic results on countable models including Ryll-Nardzewski's theorem; indiscernible sequences, Ehrenfeucht-Mostowski models; introduction to stability, rank functions, primary models, and a proof of Morley's catagoricity theorem; basic facts about infinitary languages, computation of Hanf-Morley numbers.

21-604 Introduction to Recursion Theory
Intermittent: 12 units
Models of computation, computable functions, solvable and unsolvable problems, reducibilities among problems, recursive and recursively enumerable sets, the recursion theorem, Post's problem and the Friedberg-Muchnik theorem, general degrees and r.e. degrees, the arithmetical hierarchy, the hyper-arithmetical hierarchy, the analytical hierarchy, higher type recursion. 3 hrs. lec.

21-610 Algebra I
Spring: 12 units
The structure of finitely generated abelian groups, the Sylow theorems, nilpotent and solvable groups, simplicity of alternating and projective special linear groups, free groups, the Neilsen-Schreier theorem. Vector spaces over division rings, field extensions, the fundamental Galois correspondence, algebraic closure. The Jacobson radical and the structure of semisimple rings. Time permitting, one of the following topics will be included: Wedderburn's theorem on finite division rings, Frobenius' Theorem. Prerequisite: Familiarity with the content of an undergraduate course on groups and rings. 3 hrs. lec.

21-620 Real Analysis
Fall: 6 units
A review of one-dimensional, undergraduate analysis, including a rigorous treatment of the following topics in the context of real numbers: sequences, compactness, continuity, differentiation, Riemann integration. (Mini-course. Normally combined with 21-621.) 3 hrs. lec.

21-621 Introduction to Lebesgue Integration
Fall: 6 units
Construction of Lebesgue measure and the Lebesgue integral on the real line. Fatou's Lemma, the monotone convergence theorem, the dominated convergence theorem. (Mini-course. Normally combined with 21-620.) 3 hrs. lec.

21-630 Ordinary Differential Equations
All Semesters: 12 units
Basic concepts covered are existence and uniqueness of solutions, continuation of solutions, continuous dependence, and stability. For autonomous systems, topics included are: orbits, limit sets, Liapunov's direct method, and Poincar-Bendixson theory. For linear systems, topics included are: fundamental solutions, variation of constants, stability, matrix exponential solutions, and saddle points. Time permitting, one or more of the following topics will be covered: differential inequalities, boundary-value problems and Sturm-Liouville theory, Floquet theory.

21-632 Introduction to Differential Equations
Fall: 12 units
This course serves as a broad introduction to Ordinary and Partial Differential Equations for beginning graduate students and advanced undergraduate students in mathematics, engineering, and the applied sciences. Mathematical sophistication in real analysis at the level of 21-355/356 is assumed. Topics include: essentials of Ordinary Differential Equations, origins of Partial Differential Equations, the study of model problems including the Poisson and Laplace equations, the heat equation, the transport equation, and the wave equation. 3 hrs. lec.

21-640 Introduction to Functional Analysis
Spring: 12 units
Linear spaces: Hilbert spaces, Banach spaces, topological vector spaces. Hilbert spaces: geometry, projections, Riesz Representation Theorem, bilinear and quadratic forms, orthonormal sets and Fourier series. Banach spaces: continuity of linear mappings, Hahn-Banach Theorem, uniform boundedness, open-mapping theorem. Closed operators, closed graph theorem. Dual spaces: weak and weak-star topologies (Banach-Alaoglu Theorem), reflexivity. Space of bounded continuous functions and its dual. Linear operators and adjoints: basic properties, null spaces and ranges. Compact operators. Sequences of bounded linear operators: weak, strong and uniform convergence. Introduction to spectral theory: Notions of spectrum and resolvent set of bounded operators, spectral theory of compact operators. Time permitting: Fredholm Alternative. Time permitting: Stone-Weierstrass Theorem.
Prerequisites: 21-651 and (21-720 or 21-621)

21-651 General Topology
Fall: 12 units
Metric spaces: continuity, compactness, Arzela-Ascoli Theorem, completeness and completion, Baire Category Theorem. General topological spaces: bases and subbases, products, quotients, subspaces, continuity, topologies generated by sets of functions, homeomorphisms. Convergence: nets, filters, and the inadequacy of sequences. Separation: Hausdorff spaces, regular spaces, completely regular spaces, normal spaces, Urysohn's Lemma, Tietze's Extension Theorem. Connectedness. Countability conditions: first and second countability, separability, Lindelof property. Compactness: Tychonoff's Theorem, local compactness, one-point compactification. 3 hrs. lec.

21-660 Introduction to Numerical Analysis I
Spring: 12 units
Finite precision arithmetic, interpolation, spline approximation, numerical integration, numerical solution of linear and nonlinear systems of equations, optimization in finite dimensional spaces. 3 hrs. lec.

21-690 Methods of Optimization
Fall: 12 units
An introduction to the theory and algorithms of linear and nonlinear programming with an emphasis on modern computational considerations. The simplex method and its variants, duality theory and sensitivity analysis. Large-scale linear programming. Optimality conditions for unconstrained nonlinear optimization. Newton's method, line searches, trust regions and convergence rates. Constrained problems, feasible-point methods, penalty and barrier methods, interior-point methods.

21-700 Mathematical Logic II
Spring: 12 units
Higher-order logic (type theory). Syntax, Lambda-notation, Axioms of Description and Choice, computer-assisted formal proofs, semantics, soundness, standard and non-standard models, completeness, compactness, formalization of mathematics, definability of natural numbers, representability of recursive functions, Church's Thesis. Godel's Incompleteness Theorems, undecidability, undefinability.
Prerequisites: 21-600 or 21-300

Course Website: http://gtps.math.cmu.edu/description-700.txt

21-701 Discrete Mathematics
Intermittent: 12 units
Combinatorial analysis, graph theory with applications to problems in computational complexity, networks, and other areas.

21-720 Measure and Integration
Spring: 12 units
The Lebesgue integral, absolute continuity, signed measures and the Radon-Nikodym Theorem, Lp spaces and the Riesz Representation Theorem, product measures and Fubini's Theorem.

21-721 Probability
All Semesters: 12 units
Probability spaces, random variables, expectation, independence, Borel-Cantelli lemmas. Kernels and product spaces, existence of probability measures on infinite product spaces, Kolmogorov's zero-one law. Weak and strong laws of large numbers, ergodic theorems, stationary sequences. Conditional expectation: characterization, construction and properties. Relation to kernels, conditional distribution, density. Filtration, adapted and predictable processes, martingales, stopping times, upcrossing inequality and martingale convergence theorems, backward martingales, optional stopping, maximal inequalities. Various applications of martingales: branching processes, Polya's urn, generalized Borel-Cantelli, Levy's 0-1 law, martingale method, strong law of large numbers, etc. Weak convergence of probability measures, characteristic functions of random variables, weak convergence in terms of characteristic functions. Central limit theorem, Poisson convergence, Poisson process. Large deviations, rate functions, Cramer's Theorem.
Prerequisite: 21-720

21-723 Advanced Real Analysis
Intermittent: 12 units
This course is a sequel to 21-720 (Measure and Integration). It is meant to introduce students to a number of important advanced topics in analysis.Topics include: distributions, Fourier series and transform, Sobolev spaces, Bochner integration, basics of interpolation theory, integral transforms. 3 hrs. lec.Prerequisites: 21-720 Corequisites: 21-640
Prerequisite: 21-720

21-724 Sobolev Spaces
All Semesters: 12 units
Weak derivatives, Sobolev spaces of integer order, embedding theorems, interpolation inequalities, traces.

21-732 Partial Differential Equations I
All Semesters: 12 units
An introduction to the modern theory of partial differential equations. Including functional analytic techniques. Topics vary slightly from year to year, but generally include existence, uniqueness and regularity for linear elliptic boundary value problems and an introduction to the theory of evolution equations.

21-737 Probabilistic Combinatorics
Intermittent: 12 units
This course covers the probabilistic method for combinatorics in detail and introduces randomized algorithms and the theory of random graphs. Methods covered include the second moment method, the Rödl nibble, the Lovász local lemma, correlation inequalities, martingale's and tight concentration, Janson's inequality, branching processes, coupling and the differential equations method for discrete random processes. Objects studied include the configuration model for random regular graphs, Markov chains, the phase transition in the Erdős-Rényi random graph, and the Barabási-Albert preferential attachment model.

21-738 Extremal Combinatorics
All Semesters: 12 units
Classical problems and results in extremal combinatorics including the Turán and Zarankiewicz problems, the Erdős-Stone theorem and the Erdos-Simonovits stability theorem. Extremal set theory including the Erdős-Rado sunflower lemma and variations, VC-dimension, and Kneser's conjecture. The Szemeredi regularity lemma. Algebraic methods including finite field constructions and eigenvalues and expansion properties of graphs. Shannon capacity of graphs. Chromatic number of Rn and Borsuk's conjecture. Graph decomposition including Graham-Pollack and Baranyai's theorem.

21-832 Partial Differential Equations II
Spring: 12 units
Elliptic boundary value problems, Green's theorem calculations, integral equation methods, variational formulations and Galerkin's method, regularity theory, parabolic problems and semigroups.

21-901 Masters Degree Research
All Semesters
Missing Course Description - please contact the teaching department.